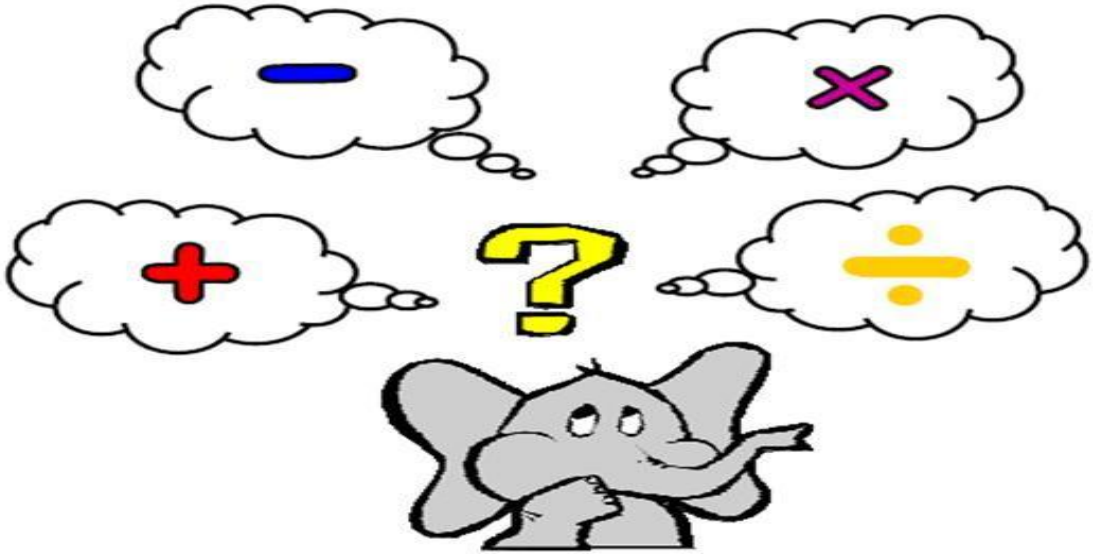
KCEPS

Approach to Calculation



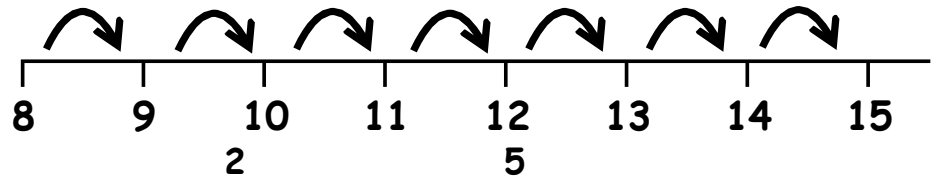
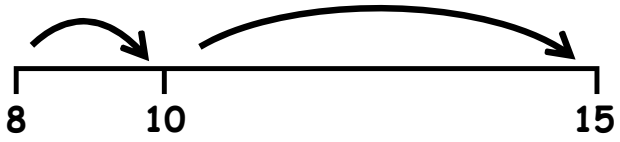
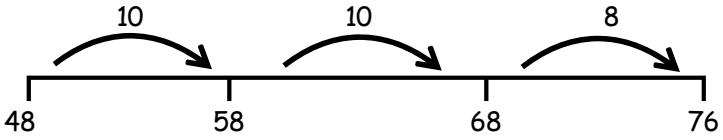
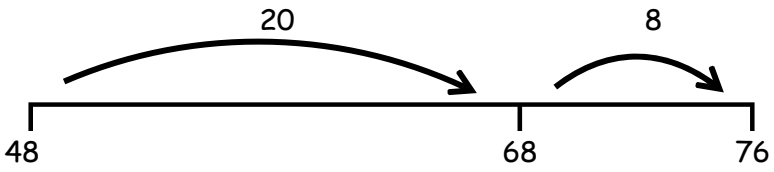
Name _____,

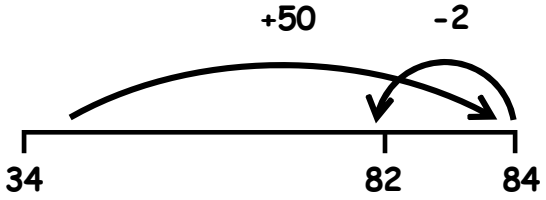
The purpose of this document is to outline the stages of progression for written calculation methods for the four number operations. Written methods of calculation are based on mental strategies, for example number bonds to 10, 20 and 100 and quick recall of times tables and associated divisions.

Each of the strategies within this document have been organised in order and it is really important to take into account the levels in which children are working so that each child has a sound understanding of the mathematics and not just a mechanical method for finding an answer. When a new strategy is introduced previous stages may need to be revisited to consolidate understanding. A sound understanding of place value and the number system is essential for children to carry out calculations efficiently and accurately.

All children should have, at their level, a reliable method for the four operations which they understand and *can explain.*

Addition

Mental Skills • Recognise the size and position of numbers • Count on in ones and tens • Know number bonds to 10 and 20 • Add multiples of 10 to any number • Partition and recombine numbers • Bridge through 10 & 100	**add** **count on** **addition** **plus** **more** **sum** **total** **altogether** **increase**
Count on using a number line Count on from the largest number Add the tens first then the units	**8 + 7 = 15**  **Or**  **28 + 48 = 76**  
Add numbers in a different order to make the calculation easier Pick out number bonds	**8 + 7 + 2 + 5 + 3** is the same as **8 + 2 + 7 + 3 + 5** **10 + 10 + 5 =25** **37 + 48 + 23** is the same as **37 + 23 + 48** **= 60 + 48** **= 108**

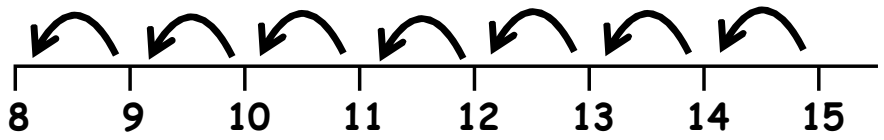
Near doubles	**8 + 9 = 17** is the same as 1 more than double 8 (16 + 1 = 17) or 1 less than double 9 (18 – 1 = 17) **37 + 36 = 73** is the same as 1 more than double 36 (72 + 1 = 73) or 1 less than double 37 (74 – 1 = 73)
Near Multiple of 10 and adjust This is a good mental strategy	**34 + 48 = 82** is the same as 34 add 50 and subtract 2  **64 + 71 = 135** is the same as 64 plus 70 plus 1 more
Partitioning TU + TU Mental strategy with jottings	**86 + 37 = 123** **86 + 37 = 123** **(80+30) + (6+7)** **110 + 13 = 123** **or** **80 + 6** **30 + 7** **110 + 13 = 123**
Expanded Column method. Add the **units** first. You should be able to explain clearly what you are doing with understanding of place value.	**47 + 76** **13** **110** **123** **648 + 286** **14** **120** **800** **934**
Column method with carrying. Carry digits are recorded **below the line**, using the words 'carry ten' or 'carry one hundred' not 'carry one'.	**47 + 76 = 123** 1 **648 + 286 = 934** 1 1 **268.74 + 86.046 + 47.8 = 402.586** 2 2 1 Extend understanding into decimals and in context of money and measures.

Subtraction

Mental Skills	count back take away
• Recognise the size and position of numbers • Count back in ones and tens • Subtract multiples of 10 from any number	fewer subtract minus less difference

Counting back using a number line

15 – 7 = 8



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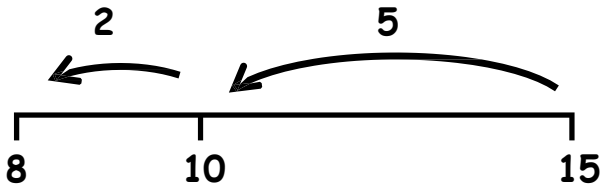
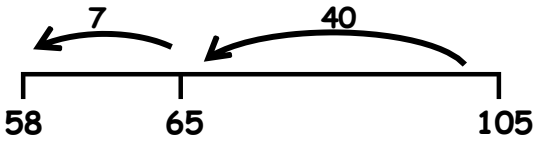
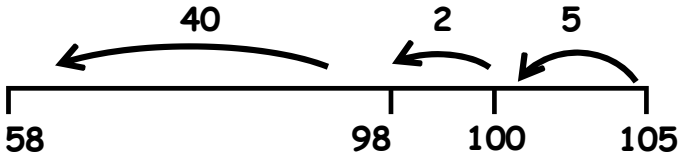


105 – 47 = 58





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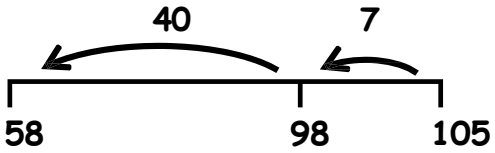
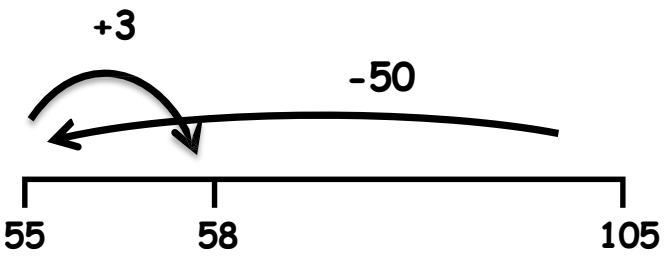


Near multiple of 10 and adjust

This is a good mental strategy

105 – 47 = 58

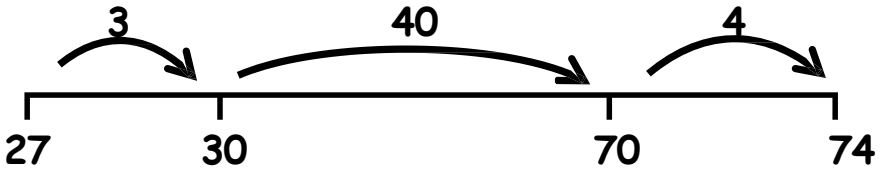
same as subtract 50 and then add back 3



Counting on using a number line or set out vertically

This method can be used with decimals but it becomes less efficient with decimals and bigger numbers

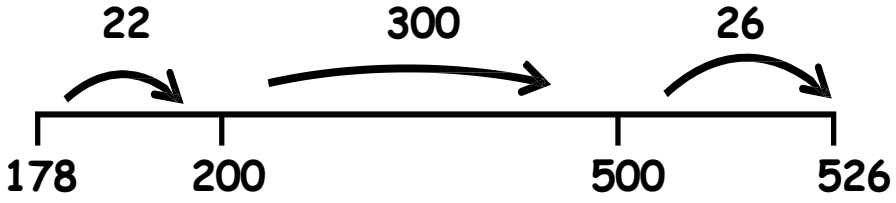
74 – 27 = 47



Leading to

$$\begin{array}{r} 74 \\ -\ 27 \\ \hline 3 \\ 40 \\ 4 \\ \hline 47 \end{array} \qquad \begin{array}{r} 30 \\ 70 \\ 74 \end{array} \qquad \begin{array}{r} 74 \\ -\ 27 \\ \hline 3 \\ 44 \\ 47 \end{array} \qquad \begin{array}{r} 30 \\ 74 \end{array}$$

526 – 178 = 348



Counting on using a number line or set out vertically

Leading to

$$\begin{array}{r} 526 \\ -\ 178 \\ \hline 22 \\ 300 \\ 26 \\ \hline 348 \end{array} \qquad \begin{array}{r} \\ \\ 200 \\ 500 \\ 526 \end{array}$$

Expanded method leading to column method

The expanded method leads to the more compact method. Only move onto this method once you have secure mental skills and you have a clear understanding of when partitioning numbers is required.

74 – 27 = 47

$$\begin{array}{rrcl} & \overset{60}{\cancel{70}} & + & \overset{14}{\cancel{4}} \\ - & 20 & + & 7 \\ \hline & 40 & + & 7 \end{array}$$

$$\begin{array}{rcc} & \overset{6}{\cancel{7}} & \overset{14}{\cancel{4}} \\ - & 2 & 7 \\ \hline & 4 & 7 \end{array}$$

Expanded method leading to column method

741 – 367 = 374

$$\begin{array}{rrrr} & \cancel{130} & & \\ & \cancel{600} & \cancel{30} & \cancel{11} \\ & \cancel{700} & \cancel{40} & \cancel{1} \\ - & 300 & 60 & 7 \\ \hline & 300 & 70 & 4 \end{array}$$

$$\begin{array}{rrrr} & \cancel{13} & & \\ & 6 & \cancel{3} & 11 \\ & \cancel{7} & \cancel{4} & \cancel{1} \\ - & 3 & 6 & 7 \\ \hline & 3 & 7 & 4 \end{array}$$

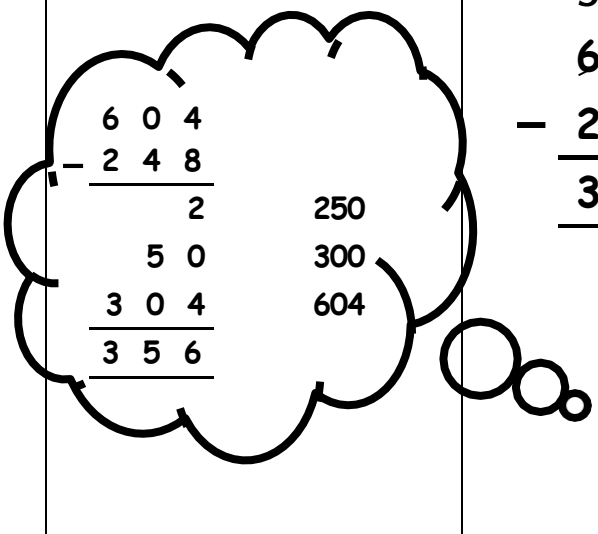
Are you choosing the best strategy for yourself?

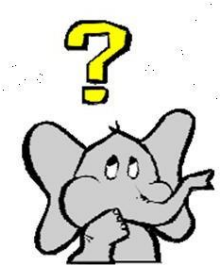
Extend column method to include zero values

604 – 248 = 356

$$\begin{array}{rrrr} & \cancel{90} & & \\ & \cancel{500} & \cancel{100} & 14 \\ & \cancel{600} & \cancel{0} & \cancel{4} \\ - & 200 & 40 & 8 \\ \hline & 300 & 50 & 6 \end{array}$$

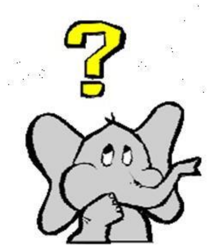
$$\begin{array}{rrrr} & 9 & & \\ & 5 & \cancel{10} & 14 \\ & \cancel{6} & \cancel{0} & \cancel{4} \\ - & 2 & 4 & 8 \\ \hline & 3 & 5 & 6 \end{array}$$



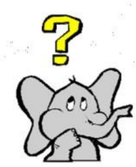


Extend column method to include decimals and zero values

Line up the decimal points
Place an extra zero if necessary



90•4 – 58•75 = 31•65

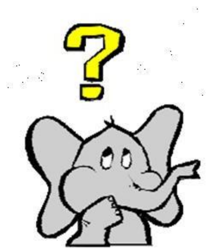


```
       9     13
  8   1̶0̶     3̶   10
  9̶   0̶  •  4̶   0
–  5   8  •  7   5
   3   1  •  6   5
```

Are you choosing the best strategy for yourself?

Choose the best strategy for yourself?

Think about each question you're doing and decide on the best strategy.



A column method may not always be the most efficient strategy.

What about 90•4 – 58•75?

Near multiple and adjust?

subtract 60 = 30•4
add back 1•25 = 31•65

Count on?
set out vertically

```
   9  0 • 4
–  5  8 • 7  5
      1 • 2  5      60
   3  0 • 4         90.4
   3  1 • 6  5
```

Multiplication

Mental skills

- Count on in different steps
- Double and halve numbers
- Recognise multiplication as repeated addition
- Quick recall of multiplication facts
- Use known facts to derive associated facts
- Multiplying by 10, 100 and 1000
- Estimation
- Multiplying by multiples of 10

multiplication **product**

once **twice** **three times**

double **groups of**

repeated addition **lots of**

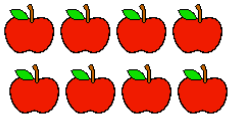
multiply **array** **row**

column **times** **multiple**

Count in groups and record as arrays

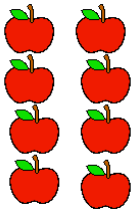
2 groups of 4 = 8

2 x 4 = 8

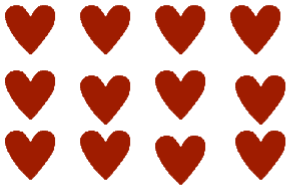


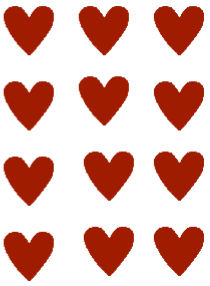
4 groups of 2 = 8

4 x 2 = 8



How many groups of 3 can be made from 12?
How many groups of 4 can be made from 12?





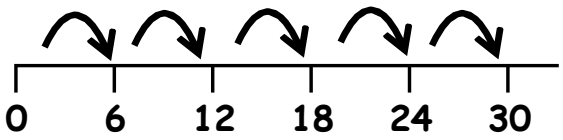
Apply times tables and associated facts to groupings

3 x 4 = 12 12 ÷ 4 = 3

4 x 3 = 12 12 ÷ 3 = 4

Count on in groups along a number line

5 x 6 = 30



Doubling and Halving

By doubling one number and halving the other you will create a question with the same answer.

Useful for finding factors of a number

5 x 16 = 80 14 x 3 = 42 15 x 8 = 120
10 x 8 = 80 7 x 6 = 42 30 x 4 = 120

Factors of 40

1 x 40 = 40
2 x 20 = 40
4 x 10 = 40
5 x 8 = 40

Multiplying by 10 and 100

Multiplying	Digits Move
x 10	1 place LEFT
x 100	2 places LEFT

25 x 10 = 250

Hundred 100	Ten 10	Unit 1
	2	5
2	5	0

25 x 100 = 2500

Thousand 1 000	Hundred 100	Ten 10	Unit 1
		2	5
2	5	0	0

9.52 x 10 = 95.2

Ten 10	Unit 1	•	Tenth 0.1	Hundredth 0.01
	9	•	5	2
9	5	•	2	

95.2 x 100 = 9520

Thousand 1 000	Hundred 100	Ten 10	Unit 1	•	Tenth 0.1	Hundredth 0.01
		9	5	•	2	
9	5	2	0	•		

Partition numbers

34 x 6 = 204

(3 0 x 6) + (4 x 6)

180 + 24 = 204

Grid multiplication

TU x U

34 x 6 = 204

x	30	4
6	180	24

180 + 24 = 204

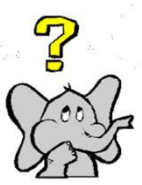
You can use the same strategy for HTU x U

Grid multiplication

TU x TU

This is a good strategy but it becomes less efficient when you move onto HTU x TU

76 x 49 = 3724



x	70	6
40	2800	240

9	630	54

Choose the easiest way to add them up?

3040 or 3430

684 294

3724 3724

1 1

Expanded vertical multiplication

TU x U

TU x TU

You can use the same strategy for HTU x U

with or without carrying

34 x 6 = 204

~~3 4~~

x 6

~~2 4~~

~~1 8 0~~

2 0 4

1

76 x 49 = 3724

~~7 6~~

x ~~4 9~~

~~5 4~~

~~6 3 0~~

~~2 4 0~~

2 8 0 0

3 7 2 4

1 1

Compact vertical multiplication

TU x U & TU x TU

HTU x U & HTU x TU

3 4

Partial carrying of digits is expected to be recorded mentally

7 6

6 8 4

3 0 4 0

3 7 2 4

1 1

× 6

2 0 4

2

× 4

9

Most able pupils extend to

- HTU x TU

Estimation is a key process with all calculations but especially important when working with bigger numbers and decimals.

274 x 32 = 8768

Estimate
300 x 30 = 9000

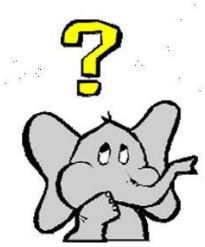
$$\begin{array}{r} 274 \\ \times\ 32 \\ \hline 548 \\ 8220 \\ \hline 8768 \end{array}$$

659 x 78 = 51402

Estimate
700 x 80 = 56000

$$\begin{array}{r} 659 \\ \times\ 78 \\ \hline 5272 \\ 46130 \\ \hline 51402 \\ 1\ \ 1 \end{array}$$

- Working with decimals



Decimals

3.4 x 7.5 = 25. 58

Estimate

3 x 8 = 24

Think of equivalent calculation

34 x 75 ÷ 100

Multiply digits together (ignore decimals)

$$\begin{array}{r} 34 \\ \times\ 75 \\ \hline 170 \\ 2380 \\ \hline 2558 \\ 1 \end{array}$$

Use estimate and equivalent calculation to decide where to place the decimal point.

2558 ÷100 = 25. 58

Division

Mental Skills

- Count back in different steps
- Double and halve numbers
- Recognise division as repeated subtraction
- Quick recall of division facts
- Use known facts to derive associated divisions
- Divide by 10, 100 and 1000
- Divide by multiples of 10

group **groups of**

lots of **divide** **divided by**

dividend **divisor** **quotient**

division **factor**

remainder **divisible**

half **halve** **share**

Key Language **18 ÷ 3 = 6**

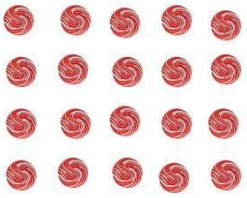
18 is the dividend
3 is the divisor
6 is the quotient

Practical examples of sharing including remainders

I have 7 sweets to share between 3 of us, what shall I do?

Understand grouping and be able to explain arrays

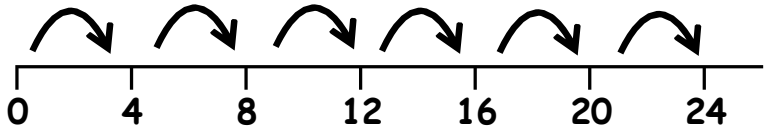
20 divided into groups of 4
gives 5 groups 20 ÷ 4 = 5



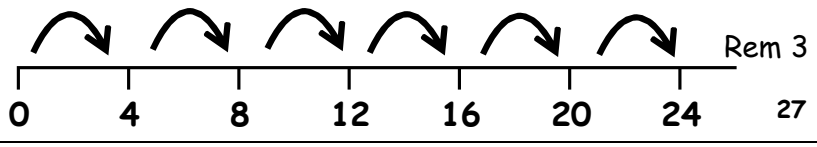
20 divided into groups of 5
gives 4 groups 20 ÷ 5 = 4

Use a number line to count on in groups

24 ÷ 4 = 6



27 ÷ 4 = 6 rem 3



Dividing by 10 or 100

Dividing	Digits Move
÷ 10	1 place RIGHT
÷ 100	2 places RIGHT

250 ÷ 10 = 25

Hundred 100	Ten 10	Unit 1
2	5	0
	2	5

2500 ÷ 100 = 25

Thousand 1 000	Hundred 100	Ten 10	Unit 1
2	5	0	0
		2	5

95.2 ÷ 10 = 9.52

Ten 10	Unit 1	Decimal point	$\frac{1}{10}$ 0.1	$\frac{1}{100}$ 0.01
9	5	.	2	
	9	.	5	2

360 ÷ 100 = 3.6

Hundred 100	Ten 10	Unit 1	.	Tenth 0.1	Hundredth 0.01
3	6	0	.		
		3	.	6	

Use known multiplication facts to work out associated divisions.

24 ÷ 4 = 6 use 4 x ? = 24

35 ÷ 7 = 5 use 7 x ? = 35

Mental division using partitioning

64 ÷ 4 = 16

(40 ÷ 4) + (24 ÷ 4)

10 + 6 = 16

102 ÷ 3 = 34

(90 ÷ 3) + (12 ÷ 3)

30 + 4 = 34

91 ÷ 7 = 13

(70 ÷ 7) + (21 ÷ 7)

10 + 3 = 13

196 ÷ 6 = 32 rem 4

(180 ÷ 6) + (12 ÷ 6)

30 + 2 32 rem 4

= 32 $\frac{4}{6}$ = 32 $\frac{2}{3}$

Chunking Up

This method is based on separating the dividend into multiples of the divisor. Initially children can split up into several chunks but with practice they should look for bigger chunks of the divisor.

64 ÷ 4 = 16

$$4\overline{)40 + 24} \quad \mathbf{10 + 6 = 16}$$

91 ÷ 7 = 13

$$7\overline{)70 + 21} \quad \mathbf{10 + 3 = 13}$$

102 ÷ 3 = 34

$$3\overline{)90 + 12} \quad \mathbf{30 + 4 = 34}$$

$$3\overline{)30 + 30 + 30 + 12} \quad \mathbf{10 + 10 + 10 + 4 = 34}$$

196 ÷ 6 = 32 rem 4

$$\mathbf{= 32\frac{4}{6} = 32\frac{2}{3}}$$

$$6\overline{)180 + 12} \quad \mathbf{30 + 2 = 32 \text{ rem } 4}$$

Short Division

64 ÷ 4 = 16

$$4\overline{)6^{2}4} \quad 1\ 6$$

91 ÷ 7 = 13

$$7\overline{)9^{2}1} \quad 1\ 7$$

102 ÷ 3 = 34

$$3\overline{)1^{1}0^{1}2} \quad 3\ 4$$

What to do with remainders. Decimal or fraction?

196 ÷ 6 = 32 rem 4

$$6\overline{)1^{1}9^{1}6} \quad 3\ 2 \text{ rem } 4 \qquad \mathbf{= 32\frac{4}{6} = 32\frac{2}{3}}$$

350 ÷ 8 = 43 rem 6

$$\mathbf{8\overline{)3\ 5^{3}0} \quad 4\ 3 \text{ rem } 6}$$

$$\mathbf{8\overline{)3\ 5^{3}0 \bullet^{6}0^{4}0} \quad 4\ 3 \bullet 7\ 5 \qquad = 43\frac{6}{8} = 43\frac{3}{4}}$$

Chunking Down

The key to efficiency with this strategy lies in the estimate that is made before the chunking starts.

e.g for 196 ÷ 6
6 x 10 = 60
6 x 20 = 120
6 x 30 = 180
6 x 40 = 240
Therefore the answer lies between 30 and 40

This method is based on subtracting multiples of the divisor. Initially children subtract several chunks but with practice they <u>should look for the biggest multiples</u> of the divisor to subtract.

102 ÷ 3 = 34

```
      34
3 / 102
     90     x30
     12
     12     x4
      0
```

196 ÷ 6 = 32 rem 4 $= 32\frac{4}{6}$ $= 32\frac{2}{3}$

```
      32
6 / 196
     60    x10
    136
     60    x10
     76
     60    x10
     16
     12    x2
      4
```

```
      32
6 / 196
    180    x30
     16
     12    x 2
      4
```

Long Division

HTU ÷ TU

List the times table you're working with to help you get started.

560 ÷ 24 = 23 rem 8 $= 23\frac{8}{24}$ $= 23\frac{1}{3}$

How many packs of 24 can we make from 560?
Estimate first
24 x 10 = 240 24 x 20 = 480 24 x 30 = 720
so the answer must be between 20 and 30

```
       23
24 / 560
     480    x20
      80
      72    x3
       8
```

24 times table	
1	24
2	48
3	72
4	96
5	120
6	144
7	168
8	192
9	216
10	240