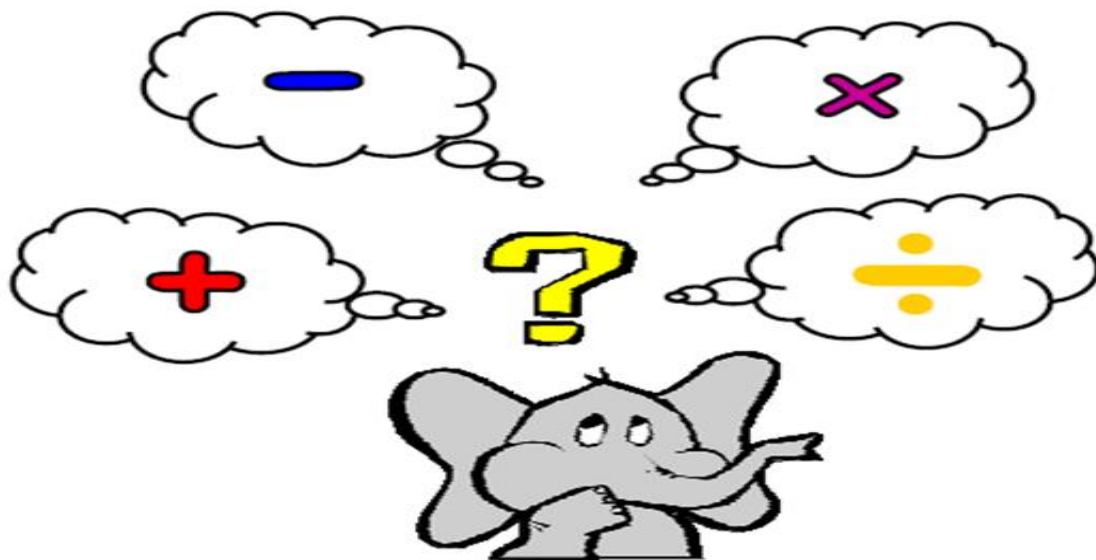# Prince William Family of Schools

Mathematics

Approach to Calculation

Name   ,

The purpose of this document is to outline the stages of progression for written calculation methods for the four number operations. Written methods of calculation are based on mental strategies, for example number bonds to 10, 20 and 100 and quick recall of times tables and associated divisions.

Each of the strategies within this document have been organised in order and it is really important to take into account the levels in which children are working so that each child has a sound understanding of the mathematics and not just a mechanical method for finding an answer. When a new strategy is introduced previous stages may need to be revisited to consolidate understanding. A sound understanding of place value and the number system is essential for children to carry out calculations efficiently and accurately.

All children should have, at their level, a reliable method for the four operations which they understand and *can explain.*

Addition

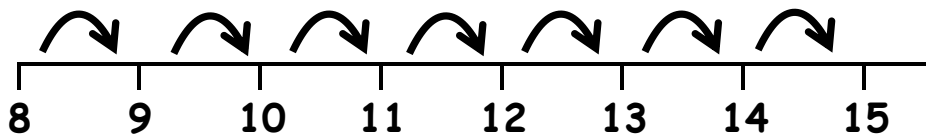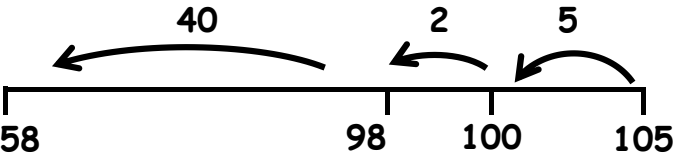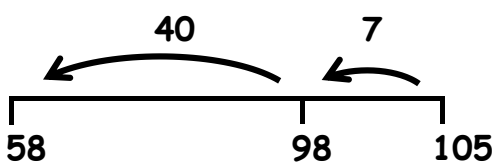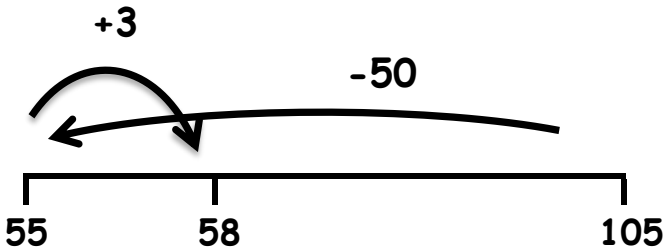|	
Mental Skills • Recognise the size and position of numbers • Count on in ones and tens • Know number bonds to 10 and 20 • Add multiples of 10 to any number • Partition and recombine numbers • Bridge through 10 & 100	**add** **count on** **addition** **plus** **more** **sum** **total** **altogether** **increase**
Count on using a number line Count on from the largest number Add the tens first then the units	**8 + 7 = 15** 8 9 10 11 12 13 14 15 2 5 Or 8 10 15 **28 + 48 = 76** 10 10 8 48 58 68 76 20 8 48 68 76
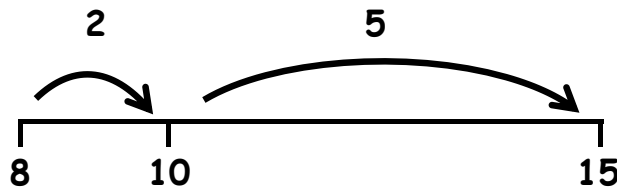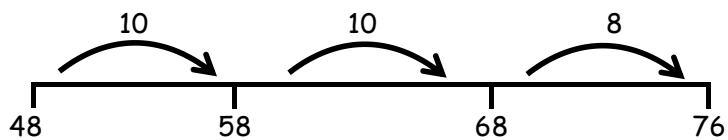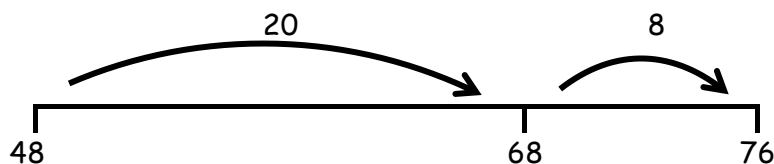
Add numbers in a different order to make the calculation easier
 	
 Pick out number bonds
	8 + 7 + 2 + 5 + 3
	 is the same as 8 + 2 + 7 + 3 + 5
	     10 + 10 + 5 =25
	 37 + 48 + 23
	 is the same as 37 + 23 + 48
	     = 60 + 48
	     = 108
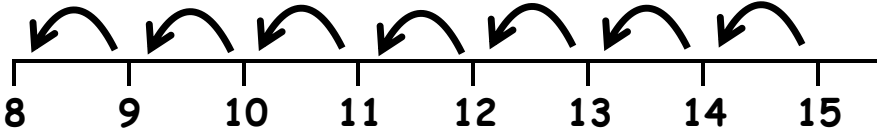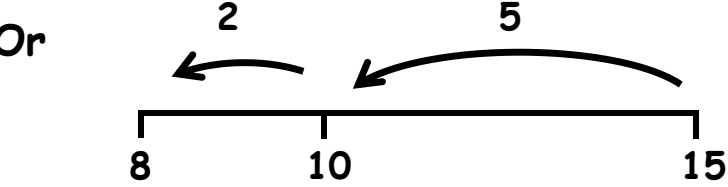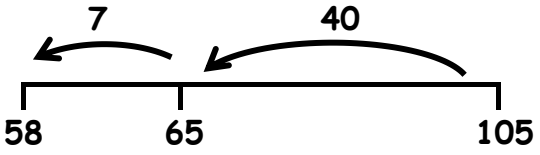
Near doubles	**8 + 9 = 17**
	 is the same as 1 more than double 8 (16 + 1 = 17)
	            or 1 less than double 9 (18 – 1 = 17)
	37 + 36 = 73
	 is the same as 1 more than double 36 (72 + 1 = 73)
	            or 1 less than double 37 (74 – 1 = 73)

| <u>Near Multiple of 10 and adjust</u>

This is a good mental strategy | **34 + 48 = 82**
 is the same as 34 add 50 and subtract 2

            +50   **-2**

  34   **82**   **84**

64 + 71 = 135   is the same as 64 plus 70 plus 1 more |

| <u>Partitioning</u>
TU + TU
Mental strategy with jottings | **86 + 37 = 123**   **or**
  86   +   37 = 123   80 + 6
(80+30) + (6+7)   30 + 7
  110   +   13   = 123   110 + 13 = 123 |

| <u>Expanded Column method</u>.
Add the **units** first.

You should be able to explain clearly what you are doing with understanding of place value. | 4 7   6 4 8
+ 7 6   + 2 8 6
   1 3   1 4
 1 1 0   1 2 0
 1 2 3   8 0 0
           9 3 4 |

| <u>Column method with carrying.</u>

Carry digits are recorded **below the line**, using the words 'carry ten'
or 'carry one hundred' not 'carry one'. | 4 7   6 4 8   2 6 8•7 4
+ 7 6   + 2 8 6   8 6•0 4 6
 1 2 3   9 3 4 + 4 7•8
   1   1 1   4 0 2•5 8 6
                 2 2 1

Extend understanding into decimals and in context of money and measures. |

Subtraction

Mental Skills	count back take away
• Recognise the size and position of numbers • Count back in ones and tens • Subtract multiples of 10 from any number	fewer subtract minus less difference

Counting back using a number line	**15 – 7 = 8**
	
	8 9 10 11 12 13 14 15
	Or
	    2 5
	8 10 15
	
	105 – 47 = 58
	      7 40
	     58 65 105
	
	    40 2 5
	    58 98 100 105
	
	Or
	    40 7
	    58 98 105

Near multiple of 10 and adjust
 	
This is a good mental strategy
	105 – 47 = 58
	same as subtract 50 and then add back 3
	
	   +3
	           –50
	55 58 105

<u>Counting on using a number line or set out vertically</u> This method can be used with decimals but it becomes less efficient with decimals and bigger numbers	**74 – 27 = 47** 27　　　30　　　　　　70　　　74 Leading to $\begin{array}{r} 74 \\ -\ 27 \\ \hline 3 \\ 40 \\ 4 \\ \hline 47 \end{array}$　$\begin{array}{l} 30 \\ 70 \\ 74 \end{array}$　　$\begin{array}{r} 74 \\ -\ 27 \\ \hline 3 \\ 44 \\ \hline 47 \end{array}$　$\begin{array}{l} 30 \\ 74 \end{array}$
Counting on using a number line or set out vertically	**526 – 178 = 348** 22　　　300　　　26 178　　200　　　　500　　526 Leading to $\begin{array}{r} 526 \\ -\ 178 \\ \hline 22 \\ 300 \\ 26 \\ \hline 348 \end{array}$　$\begin{array}{l} 200 \\ 500 \\ 526 \end{array}$
<u>Expanded method leading to column method</u> The expanded method leads to the more compact method. Only move onto this method once you have secure mental skills and you have a clear understanding of when partitioning numbers is required.	**74 – 27 = 47** 　　　　60　　　　14 　　　7̶0̶　+　4̶ 　－　20　+　7 　　　―――――――― 　　　40　+　7 　　6　1 4 　　7̶　4̶ 　－　2　7 　―――――― 　　4　7
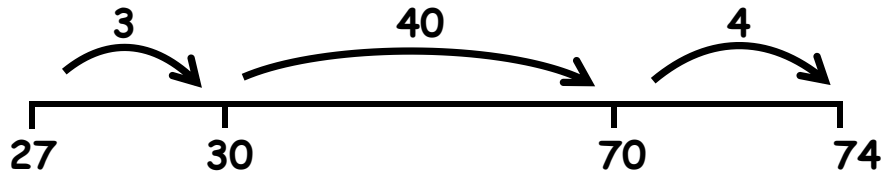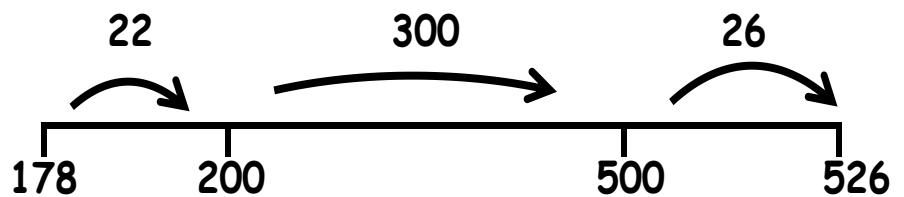
Expanded method leading to column method	$741 - 367 = 374$

$$
\begin{array}{c}
\ \ 130 \\
600 \quad\ \cancel{30}\ \ \ \ 11 \\
\cancel{700} + \cancel{40} + \cancel{1} \\
-\ \ 300 + 60 + 7 \\
\hline
\ \ \ 300 + 70 + 4 \\
\hline
\end{array}
$$

$$
\begin{array}{c}
\ 13 \\
6\ \ \cancel{3}\ \ 11 \\
\cancel{7}\ \ \cancel{4}\ \ \cancel{1} \\
-\ \ 3\ \ \ 6\ \ \ 7 \\
\hline
3\ \ \ 7\ \ \ 4 \\
\hline
\end{array}
$$

Are you choosing the best strategy for yourself?

Extend column method to include zero values	$604 - 248 = 356$

$$
\begin{array}{cc}
\begin{array}{r}
6\ 0\ 4 \\
-\ 2\ 4\ 8 \\
\hline
2 \qquad 250 \\
5\ 0 \qquad 300 \\
3\ 0\ 4 \qquad 604 \\
\hline
3\ 5\ 6 \\
\hline
\end{array}
\end{array}
$$

$$
\begin{array}{c}
\ 90 \\
500 \quad \cancel{100}\ \ \ 14 \\
\cancel{600} + \cancel{0} + \cancel{4} \\
-\ \ 200 + 40 + 8 \\
\hline
\ \ \ 300 + 50 + 6 \\
\hline
\end{array}
\qquad
\begin{array}{c}
\ 9 \\
5\ \ \cancel{10}\ \ 14 \\
\cancel{6}\ \ \cancel{0}\ \ \cancel{4} \\
-\ \ 2\ \ \ 4\ \ \ 8 \\
\hline
3\ \ \ 5\ \ \ 6 \\
\hline
\end{array}
$$
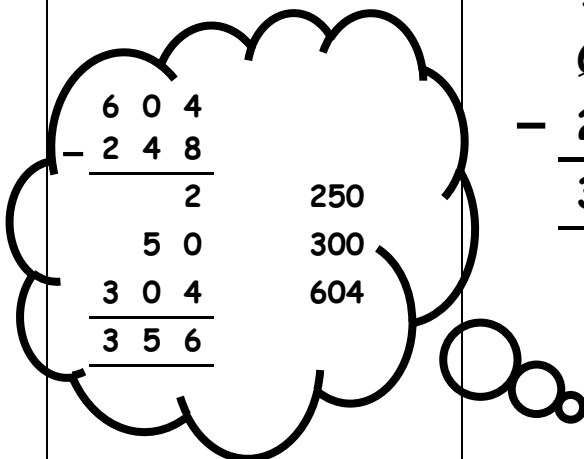
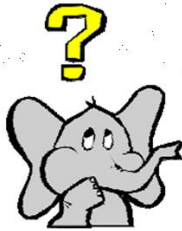|	
<u>Extend column method to include decimals and zero values</u> Line up the decimal points Place an extra zero if necessary	$90.4 - 58.75 = 31.65$ $$\begin{array}{r} 8\ \ \overset{9}{\cancel{10}}\quad\ \overset{13}{\cancel{3}}\ \ 10 \\ \cancel{9}\ \cancel{0}\ .\ \cancel{4}\ \cancel{0} \\ -\ 5\ 8\ .\ 7\ 5 \\ \hline 3\ 1\ .\ 6\ 5 \\ \hline \end{array}$$ **Are you choosing the best strategy for yourself?**
<u>Choose the best strategy for yourself?</u> Think about each question you're doing and decide on the best strategy.	A column method may not always be the most efficient strategy. **What about 90.4 – 58.75?** <u>Near multiple and adjust?</u> subtract 60 = 30.4 add back 1.25 = 31.65 <u>Count on?</u> set out vertically $$\begin{array}{r} 9\ 0\ .\ 4 \\ -\ 5\ 8\ .\ 7\ 5 \\ \hline 1\ .\ 2\ 5 \qquad 60 \\ 3\ 0\ .\ 4 \qquad 90.4 \\ \hline 3\ 1\ .\ 6\ 5 \\ \hline \end{array}$$
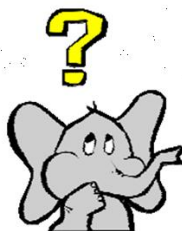
Multiplication

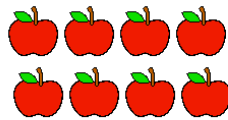| Mental skills	
- Count on in different steps - Double and halve numbers - Recognise multiplication as repeated addition - Quick recall of multiplication facts - Use known facts to derive associated facts - Multiplying by 10, 100 and 1000 - Estimation - Multiplying by multiples of 10	**multiplication product** **once twice three times** **double groups of** **repeated addition lots of** **multiply array row** **column times multiple**

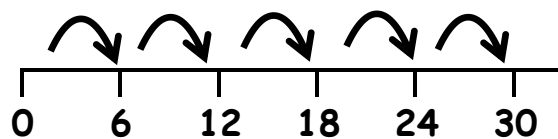| Count in groups and record as arrays	
	2 groups of 4 = 8 4 groups of 2 = 8 $2 \times 4 = 8$ $4 \times 2 = 8$ How many groups of 3 can be made from 12? How many groups of 4 can be made from 12? Apply times tables and associated facts to groupings $3 \times 4 = 12$ $12 \div 4 = 3$ $4 \times 3 = 12$ $12 \div 3 = 4$
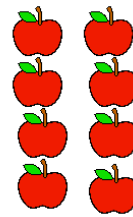
Count on in groups along a number line	
	$5 \times 6 = 30$ 0 6 12 18 24 30

<u>Doubling and Halving</u> By doubling one number and halving the other you will create a question with the same answer. Useful for finding factors of a number	$5 \times 16 = 80$ $14 \times 3 = 42$ $15 \times 8 = 120$ $10 \times 8 = 80$ $7 \times 6 = 42$ $30 \times 4 = 120$ **<u>Factors of 40</u>** $1 \times 40 = 40$ $2 \times 20 = 40$ $4 \times 10 = 40$ $5 \times 8 = 40$

<u>Multiplying by 10 and 100</u>

Multiplying	Digits Move
× 10	1 place LEFT
× 100	2 places LEFT

$25 \times 10 = 250$

Hundred 100	Ten 10	Unit 1
	2	5
2	5	0

$25 \times 100 = 2500$

Thousand 1 000	Hundred 100	Ten 10	Unit 1
		2	5
2	5	0	0

$9.52 \times 10 = 95.2$

Ten 10	Unit 1	.	Tenth 0.1	Hundredth 0.01
	9	.	5	2
9	5	.	2	

$95.2 \times 100 = 9520$

Thousand 1 000	Hundred 100	Ten 10	Unit 1	.	Tenth 0.1	Hundredth 0.01
		9	5	.	2	
9	5	2	0	.		

<u>Partition numbers</u>

$34 \times 6 = 204$

$(3\ 0\ \times\ 6)\ +\ (4\ \times\ 6)$

$180\ +\ 24\ =\ 204$

<u>Grid multiplication</u>
TU x U

You can use the same strategy for HTU x U

$34 \times 6 = 204$

×	30	4
6	180	24

$180 + 24 = 204$

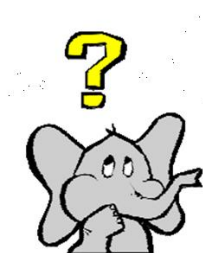|	
<u>Grid multiplication</u> TU x TU This is a good strategy but it becomes less efficient when you move onto HTU x TU	$76 \times 49 = 3724$ Choose the easiest way to add them up? | ×	70	6
---	---	---
40	2800	240

9	630	54	
			
			  3 0 4 0 or 3 4 3 0
			     6 8 4 2 9 4
			  ───────── ─────────
			  3 7 2 4 3 7 2 4
			       1 1

<u>Expanded vertical multiplication</u>
 	
TU x U
TU x TU
You can use the same strategy for HTU x U
 	
with or without carrying
	$34 \times 6 = 204$
	
	      3 4
	  × 6
	  ─────────
	      2 4
	  1 8 0
	  ─────────
	  2 0 4
	     1
	
	$76 \times 49 = 3724$
	
	         7 6
	  × 4 9
	  ──────────
	         5 4
	      6 3 0
	      2 4 0
	   2 8 0 0
	  ──────────
	   3 7 2 4
	      1 1

<u>Compact vertical multiplication</u>
TU x U   &   TU x TU
HTU x U   &   HTU x TU
Partial carrying of digits is expected to be recorded mentally
	7 6
	      3 4 × 4 9
	× 6 ──────
	───── 6 8 4
	2 0 4 3 0 4 0
	  2 ──────
	            3 7 2 4
	             1 1

Most able pupils extend to

- HTU x TU

Estimation is a key process with all calculations but especially important when working with bigger numbers and decimals.

274 x 32 = 8768   659 x 78 = 51402

Estimate   Estimate
300 x 30 = 9000   700 x 80 = 56000

```
      2 7 4              6 5 9
  ×     3 2          ×     7 8
      5 4 8          5 2 7 2
    8 2 2 0        4 6 1 3 0
    8 7 6 8        5 1 4 0 2
                    1     1
```

- Working with decimals

Decimals

$3 \cdot 4 \times 7 \cdot 5 = 25 \cdot 58$

Estimate
3 x 8 = 24

Think of equivalent calculation

$34 \times 75 \div 100$

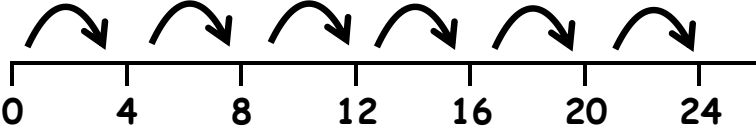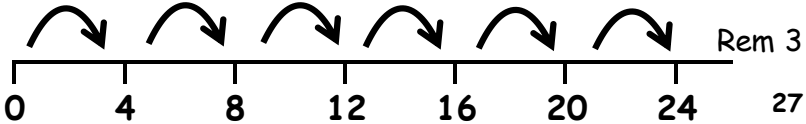Multiply digits together (ignore decimals)

```
      3 4
  ×   7 5
    1 7 0
  2 3 8 0
  2 5 5 8
      1
```

Use estimate and equivalent calculation to decide where to place the decimal point.

$2558 \div 100 = 25 \cdot 58$

Division

<u>Mental Skills</u> • Count back in different steps • Double and halve numbers • Recognise division as repeated subtraction • Quick recall of division facts • Use known facts to derive associated divisions • Divide by 10, 100 and 1000 • Divide by multiples of 10	**group** **groups of** **lots of** **divide** **divided by** **dividend** **divisor** **quotient** **division** **factor** **remainder** **divisible** **half** **halve** **share** <u>Key Language</u> $18 \div 3 = 6$ 18 is the dividend 3 is the divisor 6 is the quotient
<u>Practical examples</u> of sharing including remainders	I have 7 sweets to share between 3 of us, what shall I do?
<u>Understand grouping and be able to explain arrays</u>	 20 divided into groups of 4 20 divided into groups of 5 gives 5 groups $20 \div 4 = 5$ gives 4 groups $20 \div 5 = 4$
<u>Use a number line to count on in groups</u>	$24 \div 4 = 6$ 0 4 8 12 16 20 24 $27 \div 4 = 6 \text{ rem } 3$ Rem 3 0 4 8 12 16 20 24 27

Dividing by 10 or 100	$250 \div 10 = 25$			$2500 \div 100 = 25$			

Dividing by 10 or 100

Dividing	Digits Move
÷ 10	1 place RIGHT
÷ 100	2 places RIGHT

$250 \div 10 = 25$

Hundred 100	Ten 10	Unit 1
2	5	0
	2	5

$2500 \div 100 = 25$

Thousand 1 000	Hundred 100	Ten 10	Unit 1
2	5	0	0
		2	5

$95.2 \div 10 = 9.52$

Ten 10	Unit 1	Decimal f •	$\frac{1}{10}$ 0.1	$\frac{1}{100}$ 0.01
9	5	•	2	
	9	•	5	2

$360 \div 100 = 3.6$

Hundred 100	Ten 10	Unit 1	•	Tenth 0.1	Hundredth 0.01
3	6	0	•		
		3	•	6	

Use known multiplication facts to work out associated divisions.

$24 \div 4 = \underline{6}$ use $4 \times ? = 24$

$35 \div 7 = \underline{5}$ use $7 \times ? = 35$

Mental division using partitioning

64   ÷   4   =   16

(40 ÷ 4) + (24 ÷ 4)

10   +   6   =   16

102   ÷   3   = 34

(90 ÷ 3) + (12 ÷ 3)

30   +   4   = 34

91   ÷   7   = 13

(70 ÷ 7) + (21 ÷ 7)

10   +   3   = 13

196   ÷   6   = 32 rem 4

(180 ÷ 6) + (12 ÷ 6)

30   +   2   = 32 rem 4

$= 32\frac{4}{6}$ $= 32\frac{2}{3}$

Chunking Up	$64 \div 4 = 16$ $\qquad$ $\dfrac{10 \; + \; 6}{4\overline{)40 \; + \; 24}} = 16$
	$91 \div 7 = 13$ $\qquad$ $\dfrac{10 \; + \; 3}{7\overline{)70 \; + \; 21}} = 13$
This method is based on separating the dividend into multiples of the divisor. Initially children can split up into several chunks but with practice they <u>should look for bigger chunks</u> of the divisor.	$102 \div 3 = 34$ $\qquad$ $\dfrac{30 \; + \; 4}{3\overline{)90 \; + \; 12}} = 34$ $\dfrac{10 \; + \; 10 \; + \; 10 \; + \; 4}{3\overline{)30 \; + \; 30 \; + \; 30 \; + \; 12}} = 34$
	$196 \div 6 = 32$ rem 4 $\quad = 32\dfrac{4}{6} \quad = 32\dfrac{2}{3}$ $\dfrac{30 \; + \; 2}{6\overline{)180 \; + \; 12}} = 32$ rem 4
Short Division	$64 \div 4 = 16$ $\qquad$ $4\overline{)6\,{}^2 4}^{\,1\;6}$ $\qquad\qquad$ $7\overline{)9\,{}^2 1}^{\,1\;7}$ $91 \div 7 = 13$ $\qquad$ $3\overline{)1\,{}^1 0\,{}^1 2}^{\;3\;4}$ $102 \div 3 = 34$
	What to do with remainders. Decimal or fraction? $196 \div 6 = 32$ rem 4 $6\overline{)1\,{}^1 9\,{}^1 6}^{\;3\;2}$ rem 4 $\quad = 32\dfrac{4}{6} \quad = 32\dfrac{2}{3}$ $350 \div 8 = 43$ rem 6 $\qquad$ $8\overline{)3\;5\,{}^3 0}^{\;4\;3}$ rem 6 $8\overline{)3\;5\,{}^3 0\,.{}^6 0\,{}^4 0}^{\;4\;3\,.\;7\;5}$ $\quad = 43\dfrac{6}{8} \quad = 43\dfrac{3}{4}$

Chunking Down

The key to efficiency with this strategy lies in the estimate that is made before the chunking starts.

e.g for $196 \div 6$

$6 \times 10 = 60$
$6 \times 20 = 120$
$6 \times 30 = 180$
$6 \times 40 = 240$
Therefore the answer lies between 30 and 40

This method is based on subtracting multiples of the divisor. Initially children subtract several chunks but with practice they <u>should look for the biggest multiples</u> of the divisor to subtract.

$102 \div 3 = 34$

```
         34
3 /   102
       90        x30
       ‾‾‾
       12
       12        x4
       ‾‾‾
        0
```

$196 \div 6 = 32 \text{ rem } 4 \qquad = 32\dfrac{4}{6} \qquad = 32\dfrac{2}{3}$

```
        32                          32
6 /  196                    6/  196
      60    x10                  180    x30
     ‾‾‾                         ‾‾‾
     136                          16
      60    x10                   12    x 2
     ‾‾‾                          ‾‾
      76                           4
      60    x10
     ‾‾‾
      16
      12    x2
     ‾‾
       4
```

Long Division

HTU ÷ TU

List the times table you're working with to help you get started.

$560 \div 24 = 23 \text{ rem } 8 \qquad = 23\dfrac{8}{24} \qquad = 23\dfrac{1}{3}$

How many packs of 24 can we make from 560?
Estimate first
$24 \times 10 = 240 \qquad 24 \times 20 = 480 \qquad 24 \times 30 = 720$
so the answer must be between 20 and 30

```
          23
24 /  560
      480       x20
     ‾‾‾
       80
       72       x3
      ‾‾‾
        8
```

24 times table	
1	24
2	48
3	72
4	96
5	120
6	144
7	168
8	192
9	216
10	240