Subtraction

$$17 - 9 = 8$$

minuend - subtrahend = difference

and

Addition

$$8 + 9 = 17$$

addend + addend = sum

‘Tools to think with’

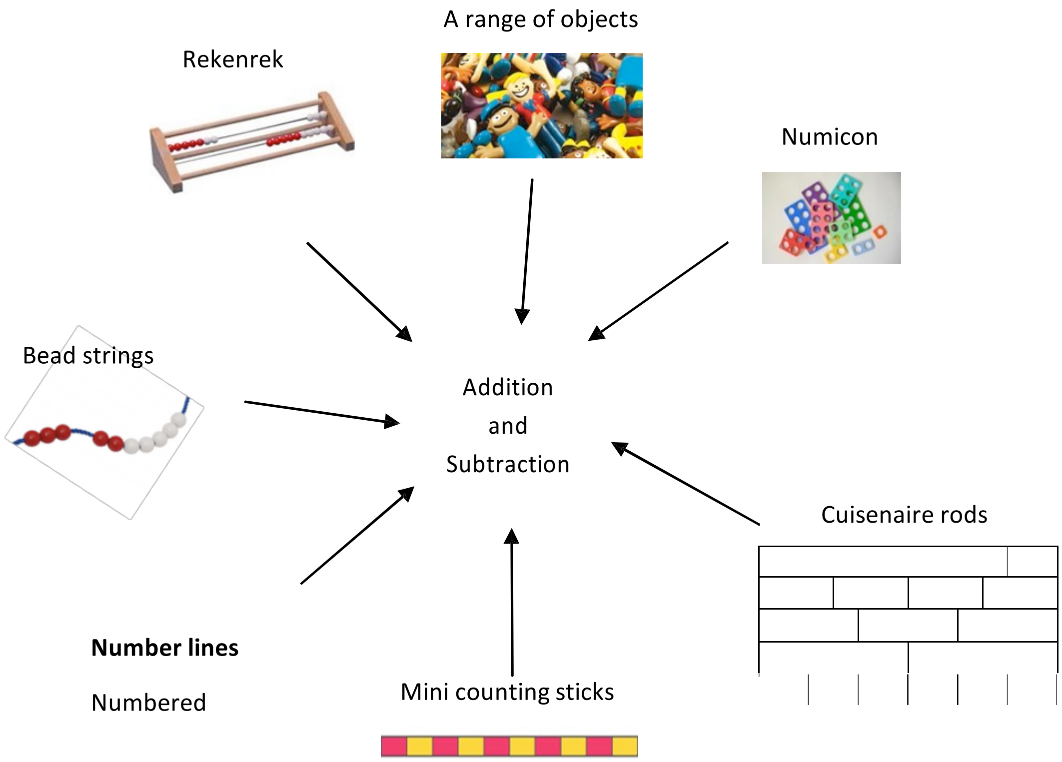
$$17 - 9 = 8$$

minuend - subtrahend = difference

Subtraction

Take away Difference

It is important that as children move through KS1 they develop their conceptual understanding of subtraction as “difference”. A concept of subtraction only as “taking away” is limited and limiting, since it relies heavily on counting and on the action of removing one amount from another and then finding the answer (what’s left) through counting. To be able to calculate with fluency and understanding, children need a range of experiences to develop their understanding of difference. Children will be unable to subtract using negative numbers unless their understanding of difference is secure.



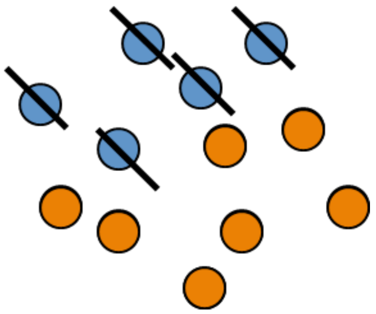
Number relationships

Partitioning sets

Children need to experience addition and subtraction using a wide range of real objects in real-life contexts as well as using unstructured (Multilink, Unifix, counters) and structured (Numicon, five rack/ten frame, Rekenrek) mathematical ‘tools to think with’.

12 - 5

Removing items from a set (reduction or take-away)



An important initial stage using real-life objects .

Issue

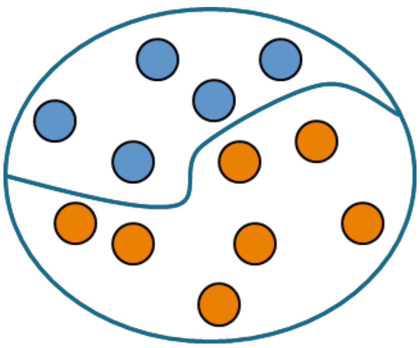
Relies on counting out the 12 objects, counting the 5 to be taken away and then counting the objects left.

If objects are actually removed from sight, children find it difficult to remember how many they started with, how many have been removed and what the counting of the objects left actually means.

Does not develop connections and fluency

12 - 5

Seeing one set as partitioned



Promotes connections and fluency

Children see that 12 is made up of 5 and 7

Helps children to see the related facts
5 + 7 = 12, 7 + 5 = 12, 12 - 7 = 5 and 12 - 5 = 7
all in the same model.

Numicon

Children need to experience addition and subtraction using a wide range of real objects in real-life contexts, as well as using unstructured (Multilink, Unifix, counters) and structured mathematical *'tools to think with'* (Numicon, five rack/ten frame, Rekenrek).

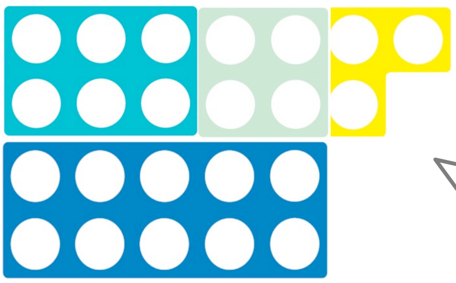
Working within 20, children take, for example, a 5 Numicon and a 3 Numicon. Then, they find the Numicon that is equal to the 5 and the 3 and say how many in total without counting.

The children can do this because they are **fluent** in their recognition of Numicon plates/shapes- they know them by their colour, they associate a quantity/number with the colour and importantly, they also know the Numicon by it's structure through 'feely bag' work - so that they can also talk about the structure of the holes in each Numicon plate/shape.

Children experience adding more that two quantities together so that they begin to develop their understanding of the associative law.

Use Numicon to calculate

6 + 3 + 4



Using Numicon - children reorganise/reorder the Numicon shapes to show that

6 + 3 + 4 = 6 + 4 + 3

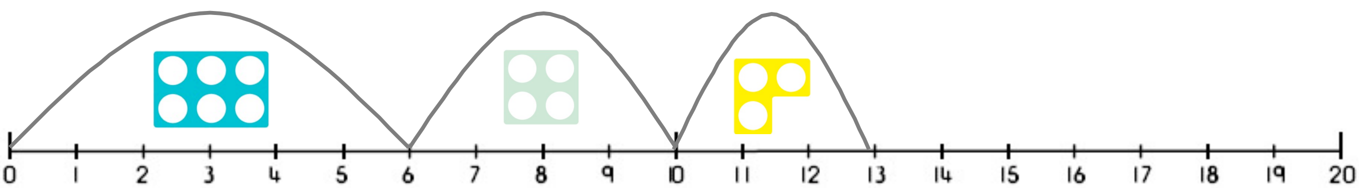
Children find the total ***without counting*** by using the 10 Numicon to show that 6 + 4 + 3 is equal to ten and some more - in this case, ten and three/thirteen.

They record sentences using the equal sign to show equality, based on what they have shown with the Numicon

6 + 3 + 4 = 6 + 4 + 3 = 10 + 3 = 13

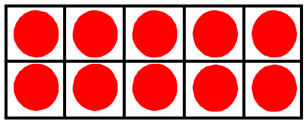
6 + 3 + 4 = 6 + 4 + 3

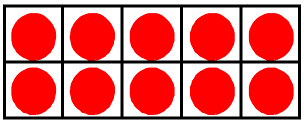
This is also shown on numbered number line and then on a blank number line.



Addition and Subtraction *within 10*

Making connections - learning number bonds





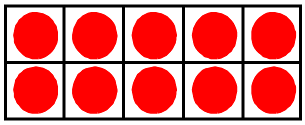
Ten Frame Addition

Resources: counters, ten frames, numeral cards 0 - 5

Children are given opportunities to count all and count on, leading to knowing how many without counting.

*Note: **No counting** can be introduced when children know that each frame represents 10 and they know how many counters are on the frame without the need to count them*

1. Place a number of counters in your frame.
2. Turn over a numeral card and add that number of counters to your frame.
3. How many counters did you start with? How many counters did you add? How many counters do you have now? Record your thinking.
4. Repeat at least 10 times.



Ten Frame Subtraction

Resources: counters, ten frames, numeral cards 0 - 10

Children are given opportunities to remove the correct number of counters , and to say and record how many are left.

*Note: **No counting** can be introduced when children know that each frame represents 10 and they know how many counters are on the frame without the need to count them*

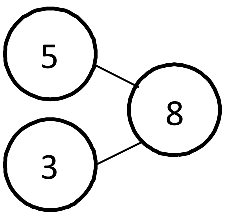
1. Place one counter in each section of your frame
2. Turn over a numeral card and remove that number of counters from your frame.
3. How many counters did you start with? How many counters did you take off? How many counters are left? Record your thinking.
4. Repeat at least 10 times.

Children can record in different ways

5 and 3 make 8

5 + 3 = 8

8 - 3 = 5

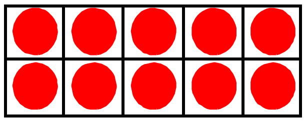


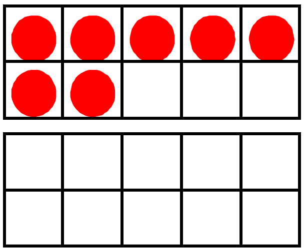
Using this model children can show the connections between addition and subtraction.

It is also a very useful model when partitioning larger numbers for addition.

Addition and Subtraction *within 20*

Unitising ten





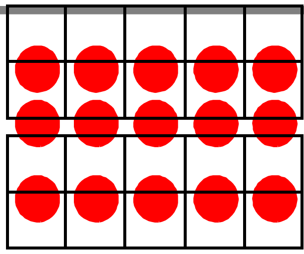
Double Ten Frame Addition

Resources: counters, ten frames, numeral cards 0 - 10

*Note: **No counting** - children should now know how many are on the five rack, ten frame without needing to count and therefore they can add the correct number of counters, and say and record how many in total without counting.*

1. Place a number of counters onto your frame (this number could be fixed - say 7 counters)
2. Turn over a numeral card and add that number of counters to your frame.
3. How many counters did you start with? How many counters did you add? How many counters are there altogether? Record your thinking.
4. Repeat at least 10 times.

To develop visualisation skills children could visualise the addition of the counters and say how many counters they would have altogether.



Double Ten Frame Subtraction

Resources: counters, ten frames, numeral cards 0 - 15

*Note: **No counting** - children should now know how many are on the five rack, ten frame without needing to count and therefore they can remove the correct number of counters, and say and record how many are left without counting.*

1. Place one counter in each of the 3 rows of your frame (15 in total).
2. Turn over a numeral card and remove that number of counters from your frame.
3. How many counters did you start with? How many counters did you take off? How many counters are left? Record your thinking.
4. Repeat at least 10 times.

To develop visualisation skills children could visualise the removal of the counters and say how many counters would be left.

Addition - mental with jottings

Using place value to partition and fluency with number bonds

6 + 3 + 4

10 + 3

23 + 38

50 + 11

16 + 5

10 + 11

87 + 35

110 + 12

17 + 18 + 13

30 + 10 + 8

OR

17 + 18 + 13

30 + 18

Partitioning numbers in different ways

Number bonds

There are very important statements in the Notes and Guidance (non-statutory) on page 11 (Year 2) and page 18 (Year 3) of the National Curriculum 2013 Programme of study - Number and place value.

In Year 2 it states that:

Pupils should partition numbers in different ways, for example

23 = 20 + 3 and 23 = 10 + 13 ***to support calculation***

They become fluent and apply their knowledge of numbers to reason with, discuss and solve problems that emphasise the value of each digit in two-digit numbers.

In Year 3 it states that:

They use larger numbers to at least 1000, applying partitioning related to place value using varied and increasingly complex problems, **building on work in year 2,** for example,

146 = 100 + 40 and 6, and 146 = 130 + 16 to support calculation

It is important that ***partitioning in different ways*** is learnt through the use of ***different*** **representations** for place value such as, straws, Dienes or Big Base.

Writing ***numbers in words also*** supports the development of this understanding.

Try watching Yeap Ban Har's video on YouTube:

Number bonds in Singapore mathematics

http://ww w.youtube.com/watch?v=9HP-8R fOju4

"Number bonds are a platform for teachers to teach students decision making, building their number sense. Under what conditions, under what situations, what actions do you take? "
(Yeap Ban Har Number bonds YouTube)

Subtraction - Difference

Using ‘tools to think with’

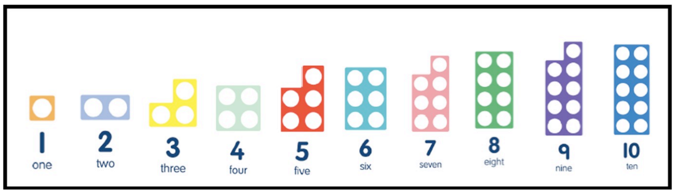
Numicon

Subtraction

Take away **Difference**

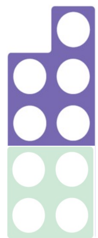
Numicon

Children need to be fluent in their recognition of all the Numicon plates/shapes (the colour, the number and the structure of the holes) in order to be able to use them for developing conceptual understanding of difference.



Find the difference…

(a game for two children)



Resources: Numicon plates, numeral cards 0 - 10

*Note: **No counting** - children must find the correct Numicon plate, and say and record the difference without counting.*

1. One child turns over a card and takes the matching Numicon plate.

2. The second child turns over a card and takes the correct Numicon plate.

3. Find the difference between the two Numicon plates by placing one on top of the other.

4, Children record the difference in their own way (do not allow children to draw around the plates). Children know how many the difference is by attending to the structure of the holes in the plate - not through colour.

5. Repeat at least 10 times

*It is important that **children use** the language of ‘the difference between’ and this will need to be modelled both orally and in writing.*

Subtraction - Difference

Using ‘tools to think with’

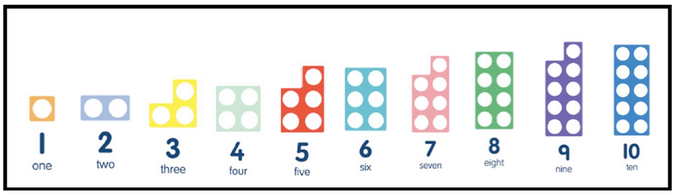
Numicon

Subtraction

Take away   **Difference**

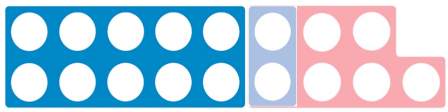
Numicon

Children need to be fluent in their recognition of all the Numicon plates/shapes (the colour, the number and the structure of the holes) in order to be able to use them for developing conceptual understanding of difference.



Exploring the difference…

What do you notice?



Resources: Numicon plates

*Note: **No counting** - children must find the correct Numicon plate and say the difference without counting.*

*t is important that **children use** the language of ‘the difference between’ and this will need to be modelled both orally and in writing.*

Take two Numicon plates - say the 7 plate and the 2 plate.

Discuss the difference (in this case 5).

Model increasing each of the original numbers by 10, by adding a ten plate to both the 7 and the 2 (so now we are finding the difference between 17 and 12) ,

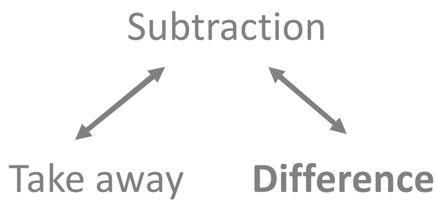
Ask children what they are finding the difference between now and what they notice?

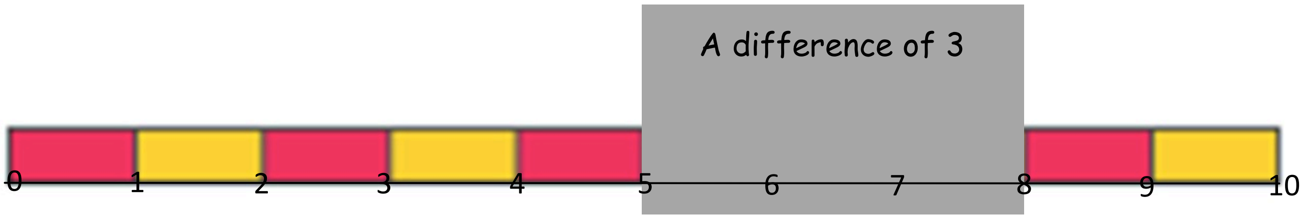
Ask **What if…**

- **What if ….**We add ten to one of the numbers?
- **What if …** We add twenty to both numbers?
- **What if ...**We add twenty to one of the numbers?

Subtraction

Counting stick - ‘The Same Difference’





This image can be ‘read’ in different ways - making the connections between addition and subtraction.

There is a difference of 3 between 5 and 8

The difference between 5 and 8 is 3

8 - 5 = 3

5 + $\square$ = 8

8 - 3 = $\square$

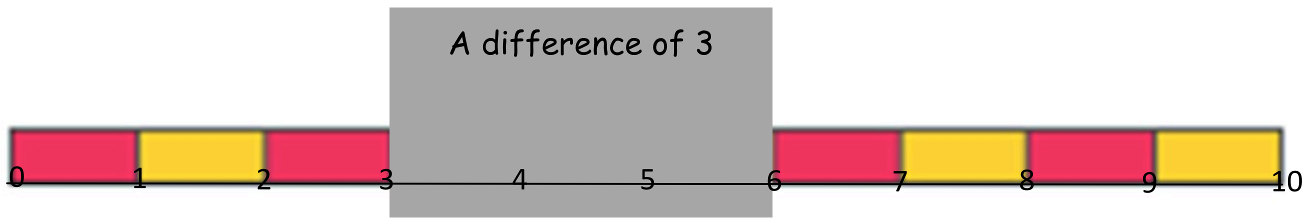
$\square$ - 3 = 5

3 + 5 = $\square$

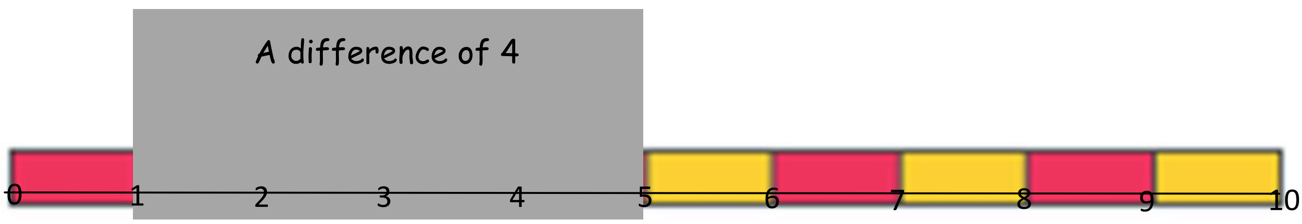
Three more than 5 is 8

Three less than 8 is 3

The slider can be moved and new statements about **a difference of three** can be made.



Make different size sliders of for children to explore the difference.

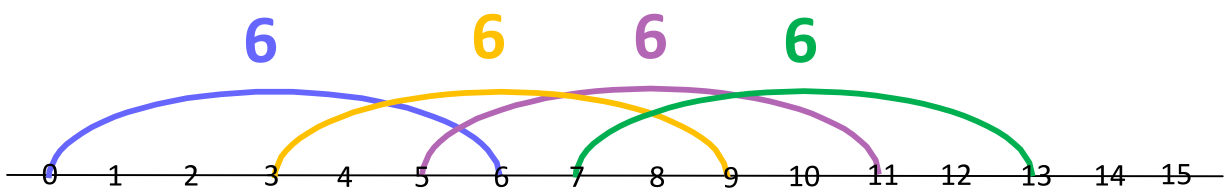


Subtraction

Number line - ‘The Same Difference’

Subtraction

Take away **Difference**



Explore and record pairs of numbers with a difference of 6:

0, 6

3, 9

5, 11

7, 13

Look at two pairs of numbers with a difference of 6

‘What do you notice?’

5, 11

7, 13

Identify other pairs through reasoning, test out your reasoning and then make a generalisation.

I have noticed that : 7 is two more than 5

and that 13 is two more than 11.

So, I think that if I add the same number to each of the numbers, then the difference between the two numbers will remain the same (6).

What if …

you subtract the same amount from each of the numbers in the calculation?

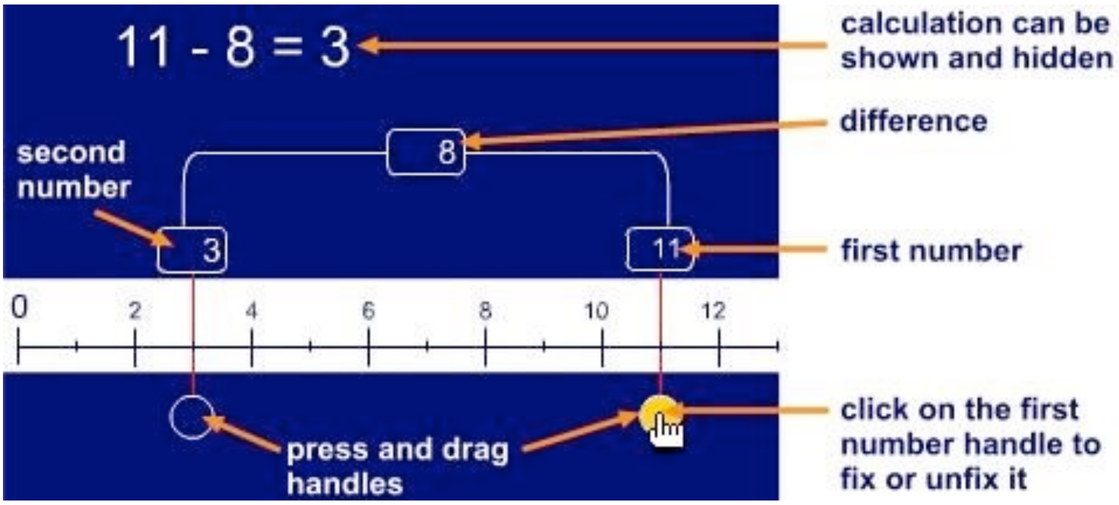
Subtraction -

Number line - 'The Same Difference'

Subtraction

Take away **Difference**

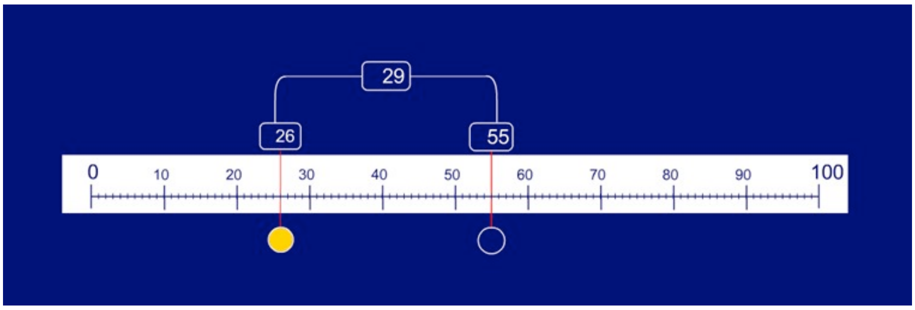
Exploring 'The Same difference' using the ITP - Number line

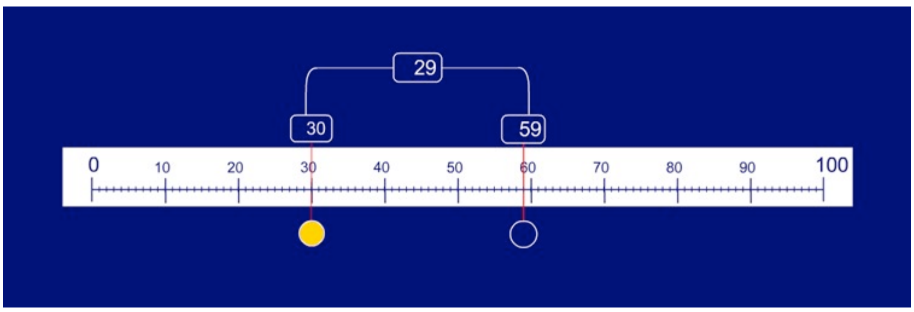


Use the ITP to explore the difference between two numbers. In this example the numbers are 55 and 26

Move each marker/handle to maintain the same difference and record at least 5 different calculations - for example:

55 - 26	26 + □ = 55
53 - 24	24 + □ = 53
50 - 21	21 + □ =50
59 - 30	30 + □ = 59
60 - 31	31 + □ = 60



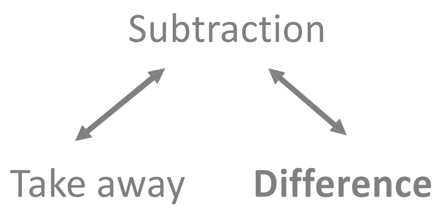


Discuss how the same difference was maintained.

Decide which calculation is the easiest to calculate mentally and why.

Subtraction -

'The Same Difference'



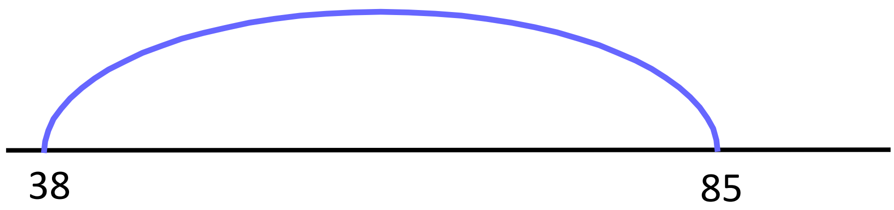
'The Same difference' Ian Sugarman

Ian Sugarman suggests that this mental strategy is a "more child-friendly alternative to decomposition" (Sugarman 2007).

It is based on children understanding what they are doing and the application of their number sense; their knowledge, understanding and application of number and operations.

"The algorithm involves a transformation of the given 'awkward' numbers into a pair that is much easier to work with...The logic behind this algorithm can be graphically modelled to pupils as a rectangle which highlights the difference between two numbers on a number line" (Sugarman, 2007).

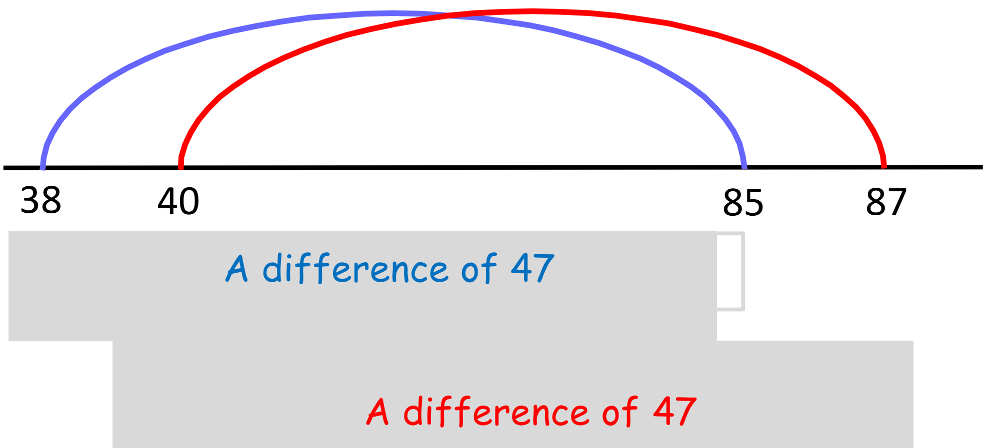
85 - 38



85 - 38

+2 +2

87 - 40



85 - 38 = 87 - 40 = 47

Subtraction - Difference

Informal written method

‘The Same Difference’ Ian Sugarman, 2007

See Ian Sugarman article in Appendix.

Subtraction

Take away **Difference**

17 - 9 = 8

minuend - subtrahend = difference

‘The Same difference’

Ian Sugarman, 2007 states the following:

“ All the examples I modelled followed a common pattern. The subtrahend was always a number ending in 9, 8, 7, or 6 so that it was always greater than the unit digit of the minuend. In every case, it was the subtrahend that was rounded up to the next decade. This constraining of the range of options seemed to be a sensible teaching strategy to adopt at the first exposure to the algorithm.”

Two-digit **-** two-digit	Three-digit **-** two-digit
53 - 28	182 - 56
72 - 38	173 - 48
55 - 26	165 - 37
63 - 27	191 - 38
84 - 36	174 - 56
73 - 29	181 - 47
81 - 57	174 - 57
52 - 28	173 - 48

Subtraction - Difference

Informal written method

'The Same Difference' (Ian Sugarman, 2007)

Subtraction

Take away **Difference**

17 - 9 = 8

minuend - subtrahend = difference

'The Same difference'

As children develop their skills of using the 'same difference' we need to ensure that the children are flexible and can make decisions about which number will be rounded - the minuend or the subtrahend, and whether to round up or round down.

Children will need planned opportunities, using well designed questions, where they round up the minuend and the subtrahend, and round down the minuend and subtrahend and decide which results in the most efficient calculation for them.

This will support the development of fluency with understanding.

84 - 36

	Round up	Round down
Round the **minuend**	90 - 42	80 - 32
Round the **subtrahend**	88 - 40	78 - 30

For most children and adults this is likely to be the favoured transformation.

Try with other calculations

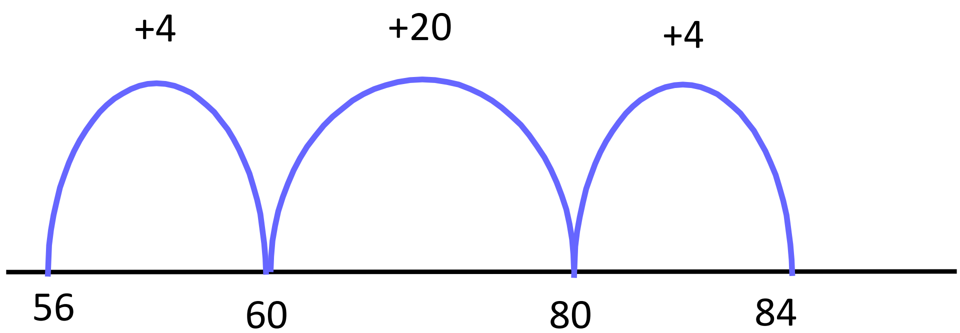
Subtraction - Difference

‘Counting up to find the difference”

Counting up from the smaller to the larger number

(from the subtrahend to the minuend)

84 - 56



Children will need to have a sound understanding of the concept of “finding the difference”.

Children must experience finding different missing numbers in a variety of subtraction calculations.

$17 - 9 = \square$ $13 - \square = 4$ $\square - 6 = 12$

Children need to know that...

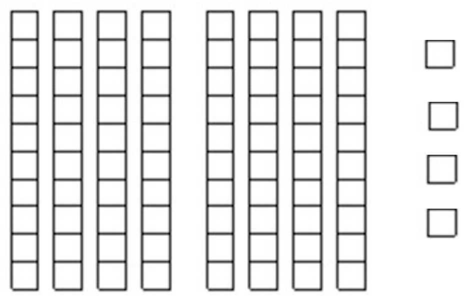
$56 + \square = 84$ ...is the same as... $84 - 56 = \square$

Subtraction - (two-digit subtract two-digit)

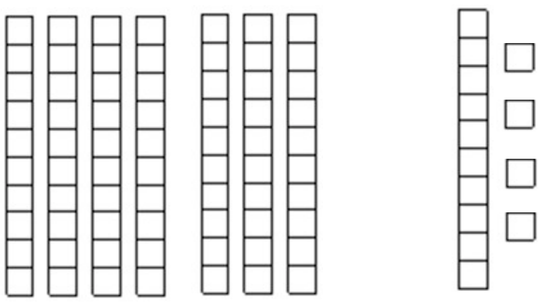
Modelling using base 10 resources (Dienes or straws)

84 - 56

When using base 10 (Dienes) resources for subtraction of 84 - 56 we only show 84.



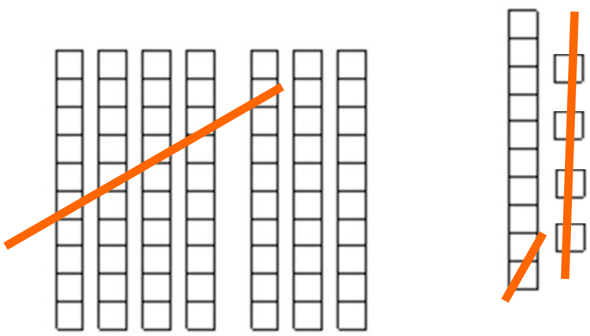
84 - 56



84 - 56

70 14

Use Number sense and knowledge of *partitioning in different ways* to decide how to partition 84.



84 - 56

70 14

Subtract 50 from the 70 and 6 from the 14 mentally.

$$\begin{array}{rr} 70 & 14 \\ -\ 50 & 6 \\ \hline 20 & 8 \end{array}$$

Leading to

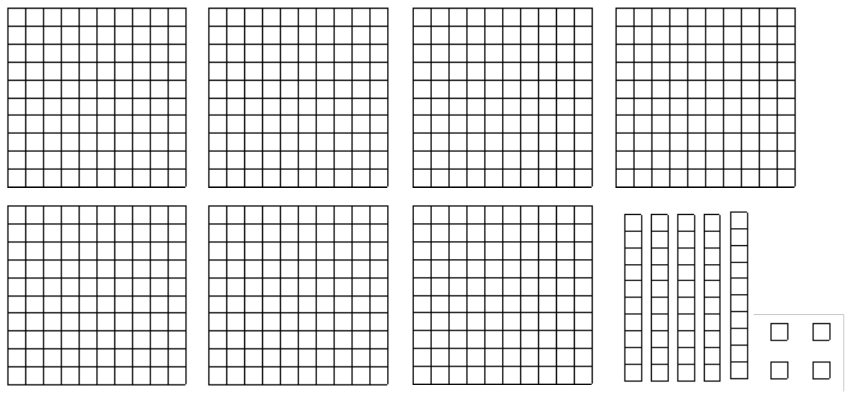
$$\begin{array}{r} \overset{7}{\cancel{8}}{}^{1}4 \\ -\ 56 \\ \hline 28 \end{array}$$

Subtraction (three digit subtract three-digit)

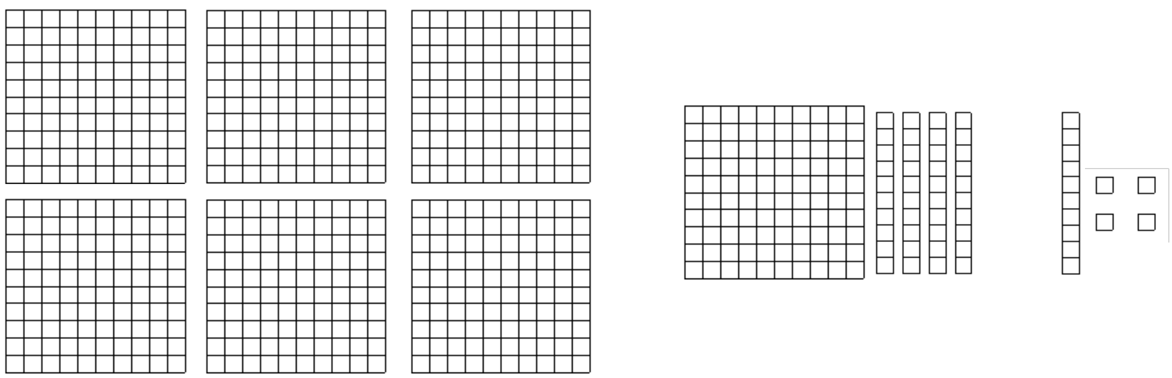
Using base 10 resources

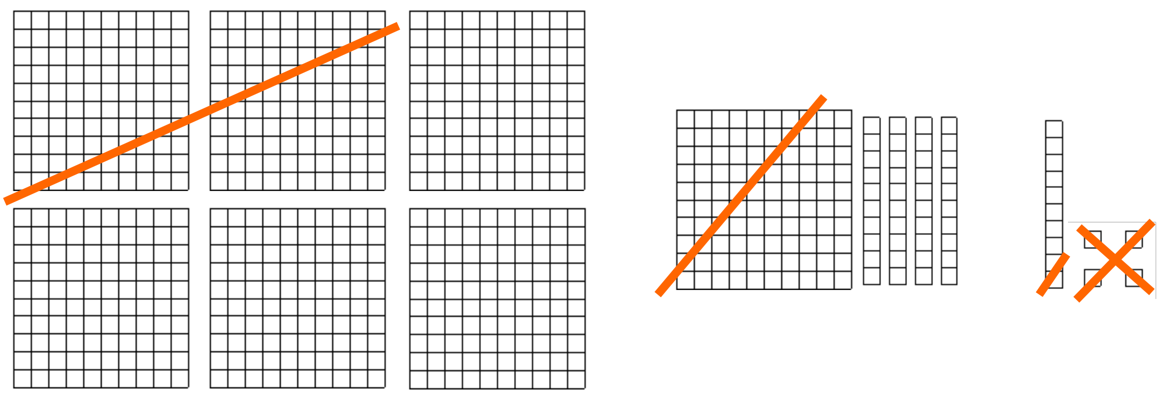
Expanded Decomposition for 754 - 286

Model using Base 10 representations (Dienes or straws)



754 - 286





7 5 4

Subtraction

Subtraction

Take away **Difference**

Expanded decomposition using base 10

Decomposition

Expanded Decomposition

$$\begin{array}{rcl} 754 & = & 700 + 50 + 4 \\ -\ \underline{286} & & \underline{200 + 80 + 6} \end{array}$$

$$\begin{array}{rl} = & 700 + 40 + 14 \\ & \underline{200 + 80 + 6} \end{array}$$

$$\begin{array}{rl} = & 600 + 140 + 14 \\ & \underline{200 + 80 + 6} \\ & 400 + 60 + 8 \quad = 468 \end{array}$$

Formal written method - Decomposition

6 14 1

$$
\begin{array}{r}
- \quad 2\ 8\ 6 \\
\hline
\\
\hline
4\ 6\ 8 \\
\hline
\\
\hline
\end{array}
$$

Subtraction - Difference

‘Counting up to find the difference”

Subtraction

Take away   **Difference**

Counting up from the smaller to the larger number

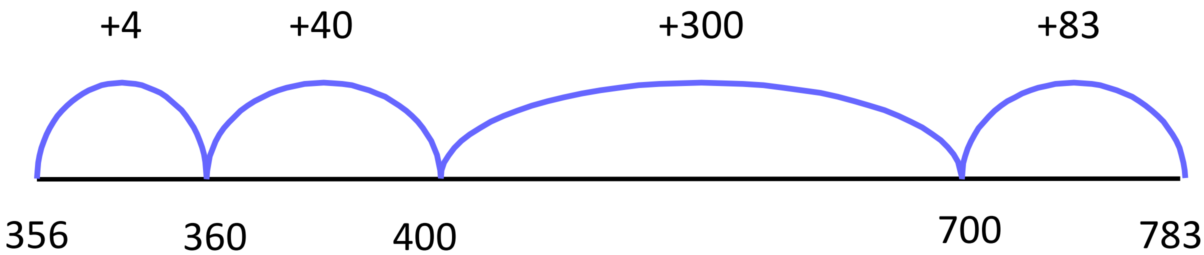
Noticing

Making links between efficient counting up on a number line and

a formal written method for subtraction.

What do you notice?

783 - 356



	783	
-	356	
	4	to 360
	40	to 400
	300	to 700
	83	to 783
	427	

Subtraction

Formal written method - decomposition

Subtraction

Take away **Difference**

Progression in exchanging

Exchange of units needed

$$\begin{array}{r} 322 \\ -\ 117 \\ \hline \\ \hline \end{array}$$

Exchange of tens needed

$$\begin{array}{r} 356 \\ -\ 182 \\ \hline \\ \hline \end{array}$$

Exchange of tens and units

$$\begin{array}{r} 311 \\ -\ 133 \\ \hline \end{array}$$

Exchange of hundreds

Zero in minuend (tens)

$$305$$

Zero in minuend (tens)

Exchange needed in hundreds, tens and units

$$\begin{array}{r} 303 \\ -\ 124 \\ \hline \end{array}$$

Zero in minuend (hundreds)

Exchange needed in tens and units and then thousands and hundreds

$$\begin{array}{r} 2072 \\ \hline -\ 1553 \end{array}$$

Zero in minuend (hundreds)

Exchange needed in thousands, hundreds, tens and units

$$\begin{array}{r} 2072 \\ \hline -\ 1583 \end{array}$$

$$\begin{array}{r} -\ 124 \\ \hline \end{array}$$

Zero in minuend
(hundreds and tens)

Exchange needed in
thousands, hundreds, tens
and units

3

0

0

8

- 1439

Addition - expanded method using knowledge of place value

$$67 + 25 = 92$$

addend + addend = sum

Expanded vertical method

Add ones first, then tens, then hundreds

Addition of ones crosses into the tens

$$\begin{array}{r} 67 \\ +\ \ 25 \\ \hline 12 \\ 80 \\ \hline 92 \\ \hline \end{array}$$

$$\begin{array}{r} 158 \\ +\ \ 37 \\ \hline 15 \\ 80 \\ 100 \\ \hline 195 \\ \hline \end{array}$$

Expanded vertical method

Addition of tens crosses into the hundreds

$$\begin{array}{r} 282 \\ +\ \ 31 \\ \hline 3 \\ 110 \\ 200 \\ \hline 313 \\ \hline \end{array}$$

Expanded vertical method Addition of ones crosses into the tens Addition of tens crosses into the hundreds

$$\begin{array}{r} 164 \\ +\ \ 177 \\ \hline 11 \\ 130 \\ 200 \\ \hline 341 \\ \hline \end{array}$$

Addition - expanded method using knowledge of place value and mental calculation

$$67 + 25 = 92$$

addend + addend = sum

Expanded vertical method

three-digit + three-digit

Addition of ones crosses into the tens

Addition of tens crosses into the hundreds

Addition of hundreds crosses into thousands

$$\begin{array}{r} 758 \\ +\ 675 \\ \hline 13 \\ 120 \\ 1300 \\ \hline 1433 \\ \hline \end{array}$$

Expanded vertical method

four-digit + three-digit

Addition of ones crosses into the tens

Addition of tens crosses into the hundreds

Addition of hundreds crosses into thousands

$$\begin{array}{r} 2635 \\ +\ 897 \\ \hline 12 \\ 120 \\ 1400 \\ 2000 \\ \hline 3532 \\ \hline \end{array}$$

Using the formal written method for addition

Experience of lots of addition questions, with NO ‘carrying’, using a formal written method, can lead to misconceptions.

If children have reached the stage where they are being asked to calculate using a ‘formal written method’, where there is NO ’carrying, *then it must be the case* that they have all the skills, knowledge and understanding necessary to carry out these calculations mentally, particularly if they are two-digit add two-digit calculations!

The language of ‘carrying’

$$\begin{array}{r} 379 \\ +\quad 54 \\ \hline 433 \\ \hline {\scriptstyle 1\ 1\ \ } \end{array}$$

9 add 4 equals 13

put the 3 in the units column and “carry the 1” .

(Children need to have mathematical understanding of what “carry the 1” actually means.)

Repeating a set of procedural steps, with the procedural language is not necessarily evidence of understanding.

247 + 35 = 282

addend + addend = sum

'Carrying' from the units to the tens

$$\begin{array}{r} 247 \\ + \\ \underline{35} \\ 282 \\ \hline {\scriptstyle 1} \end{array}$$

$$\begin{array}{r} 1525 \\ + \quad 68 \\ \hline 1593 \\ \hline {\scriptstyle 1} \end{array}$$

'Carrying' from the tens to the hundreds

$$\begin{array}{r} 282 \\ + \quad 31 \\ \hline 313 \\ \hline {\scriptstyle 1} \end{array}$$

$$\begin{array}{r} 1432 \\ + \quad 84 \\ \hline 1516 \\ \hline {\scriptstyle 1} \end{array}$$

$$\begin{array}{r} 1043 \\ + \quad 85 \\ \hline 1128 \\ \hline {\scriptstyle 1} \end{array}$$

247 + 35 = 282

addend + addend = sum

‘Carrying’

from the units to the tens **and**

from the tens into the hundreds

$$\begin{array}{r} 379 \\ +\ \ 54 \\ \hline 433 \\ \hline {}^{1\ 1} \end{array}$$

$$\begin{array}{r} 1676 \\ +\ 156 \\ \hline 1832 \\ \hline \end{array}$$

$$\begin{array}{r} {}^{1\ 1} \\ 1208 \\ +\ 295 \\ \hline 1503 \\ \hline \end{array}$$

‘Carrying’

into a new column

$$\begin{array}{r} 965 \\ +\ \ 51 \\ \hline 1016 \\ \hline {}^{1} \end{array}$$

$$\begin{array}{r} 731 \\ +\ 652 \\ \hline 1383 \\ \hline {}^{1} \end{array}$$

‘Carrying’

from the units to the tens and

from the tens into the hundreds

and into a new column

$$\begin{array}{r} 989 \\ +\ \ 69 \\ \hline 1058 \\ \hline {}^{1\ 1} \end{array}$$

247 + 35 = 282

addend + addend = sum

Adding 3 or more integers

With ‘carrying’

from the units to the tens and

from the tens into the hundreds

$$\begin{array}{r} 52 \\ 417 \\ +\ 34 \\ \hline 503 \\ \hline {\scriptstyle 1\ 1} \end{array}$$

$$\begin{array}{r} 378 \\ 102 \\ +\ 523 \\ \hline 1003 \\ \hline {\scriptstyle 1\ 1} \end{array}$$

‘Carrying’

the ‘carrying’ digit is more than one

$$\begin{array}{r} 75 \\ 88 \\ +\ 769 \\ \hline 932 \\ \hline {\scriptstyle 2\ 2} \end{array}$$

‘Carrying’

the ‘carrying’ digit is more than one

$$\begin{array}{r} 94 \\ 139 \\ 358 \\ +\ 562 \\ \hline 1153 \\ \hline {\scriptstyle 2\ 2} \end{array}$$

‘Carrying’

Using money and decimal numbers

$$\begin{array}{r} £86.74 \\ £33.57 \\ +\ £0.82 \\ \hline £121.13 \\ \hline 1\ 2\ \ 1 \end{array}$$

$$\begin{array}{r} 77.65 \\ +\ 8.87 \\ \hline 86.52 \\ \hline 1\ 1\ \ 1 \end{array}$$

Subtraction

Extending to decimal numbers

Subtraction

Take away **Difference**

Extending methods to include decimal numbers

Methods used should also be applied to a range of decimal numbers, including measurements.

Children's understanding of place value must be at a sufficient level for them to use decimal numbers effectively within calculations.

Find the difference between two three-digit sums of money, with and without adjustment from the pence to the pounds.

£8.95 - £4.38

£7.50 - £2.84

Find the difference between two decimal fractions containing the same number of decimal places, with up to three digits.

9.42 - 6.78

72.5km - 4.6km

Find the difference between decimal fractions containing either one or two decimal places, with up to three digits.

324.9 - 7.25

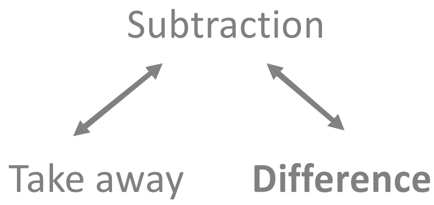
14.24 - 8.7

Time

Difference should be calculated on a number line

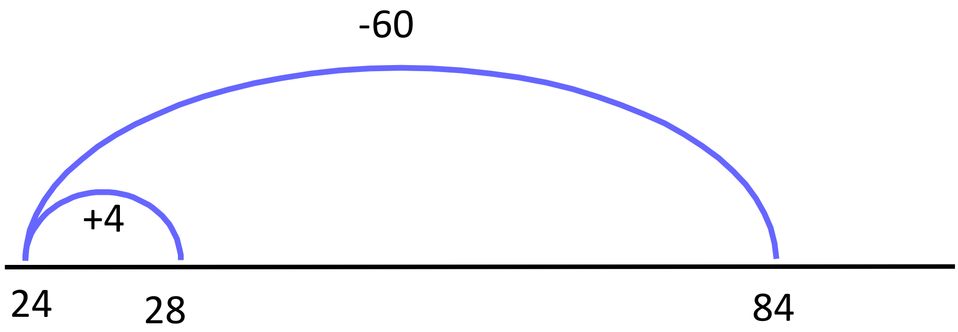
Subtraction - Compensation

Subtract the next multiple of 10 and adjust



Compensation

84 - 56



Subtraction

Making decisions

Subtraction

Take away → Subtraction ← **Difference**

It is important that children **always** look at a calculation and make decisions about which strategy to use.

Children need to ask themselves	
Can I do this in my head? →	Use a mental strategy
Can I do this in my head with jottings? →	Use a mental strategy and make jottings
Should I use an informal written method? →	Use an informal written method
Should I use a formal written method? →	Use a formal written method

Sort these calculations into the categories above and discuss with your partner.

Then calculate and discuss the decisions you made.

176 - 40	84 - 56
2001 - 1997	342 - 157
385 - 165	815 - 278
555 - 99	421 - 397
3005 - 1998	604 - 288
264 - 49	303 - 117

I would do this mentally, in my head
I would do this mentally with jottings
I would use an informal written method
I would use a formal written method

176 - 40	84 - 56
2001 - 1997	342 - 157
385 - 165	815 - 278
555 - 99	421 - 397
3005 - 1998	604 - 288
264 - 49	303 - 117

Appendix

Subtraction using:

Rekenrek: Take away’

Rekenrek: Difference

Rekenrek: One more, one less

Cuisenaire: Difference

Negative Number strategy

ITP - Interactive Teaching Programs: Difference

Making decisions about subtraction:

Appendix - page 1

Subtraction - Take away

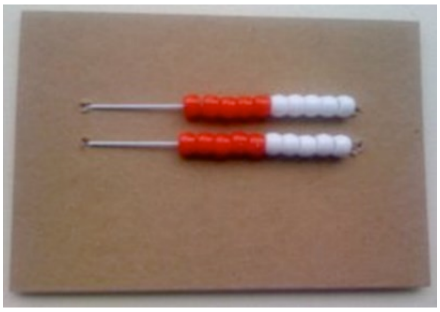
Using ‘tools to think with’ - Rekenrek

Subtraction

Take away Difference

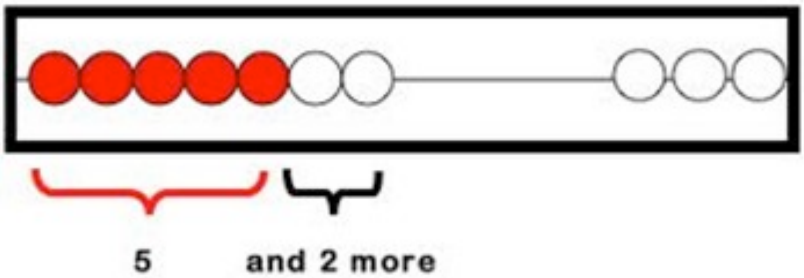
The Rekenrek

Here are some rules about using a Rekenrek:



> The beads are not counted, the children are encouraged to subitise (to know how many without counting).
>
> The starting position should show all beads pushed to the far right.
>
> Children enter the number by sliding beads to the left in a one-push motion.

Using one row on the Rekenrek - the other row is covered with paper or a cloth



Seven is seen as “5 and 2 more”

This model allows children to develop their conceptual understanding of addition and subtraction at the same time. They compose and decompose numbers, developing fluency with understanding.

The beads on the Rekenrek above show:

> That seven is made up of 5 and 2 more
>
> That seven is made up of 2 and 5 more
>
> That ten is made up of 5 and 2 and 3 or 7 and 3 or 3 and 7…
>
> That 10 take away 7 is 3
>
> The three beads on the right can be covered and the children are asked to say how many are hidden and how they know.

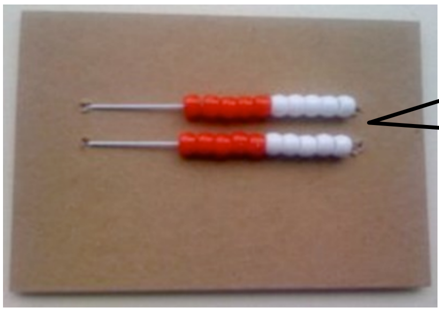
Appendix - page 2

Subtraction - Take away

Using ‘tools to think with’ - Rekenrek

Subtraction

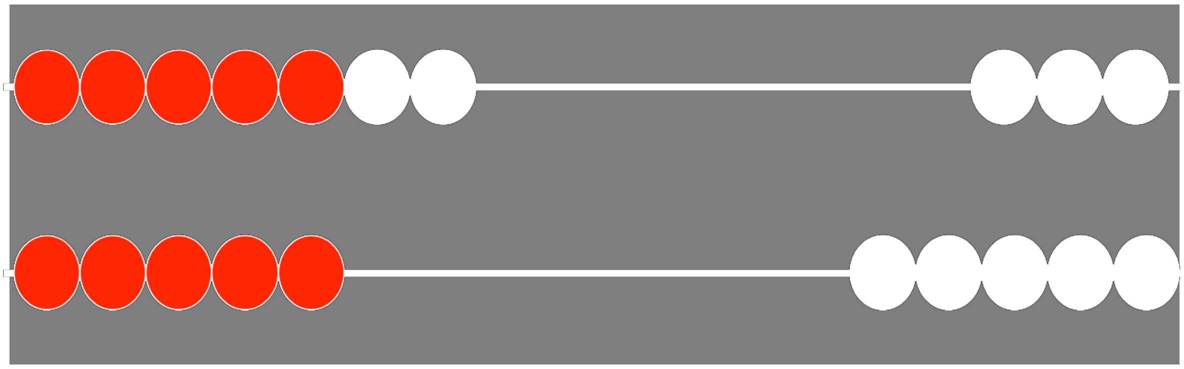
Take away Difference



The starting point - shows all the beads pushed to the far right.

Enter the number by sliding beads to the left in a one-push motion (subitise - no counting)

Using both rows on the Rekenrek



This model allows children to develop their conceptual understanding of addition and subtraction at the same time. They compose and decompose numbers, developing fluency with understanding.

The beads on the Rekenrek above show that:

Ten can be 5 red beads and 5 white beads

Ten can also be 5 red bead on one row and 5 red beads on the other row

Twelve is 10 reds beads and 2 white beads

Ten is 7 and 3

Ten is 5 and 5

Twenty is... 12 and 8 / 8 and 12 / 5 and 5 and 2 and 3 and 5 / 7 and 3 and 5 and 5...

Twenty take away 12 is 8 / twenty take away 8 is 12 …

Seven is 2 more that 5 / 5 is 2 less than 7 …

The difference between 7 and 5 is 2...

Subtraction - Difference

Using 'tools to think with' - Rekenrek

Subtraction

Take away **Difference**

Rekenrek

This Rekenrek shows:

The difference between 10 and 3 is 7

Children could also state the following:

7 is 3 less than 10

A difference of...

Move 3 beads to the left of the Rekenrek on the top row

As children to move beads on the bottom row so that there is a difference of 2

How many different ways can you make a difference of 2?

Try other numbers.

Find the difference...

(a game for two children)

Resources: a set of 1 to 10 cards, one Rekenrek between 2 children

1. One child turns over a card and slides that number of beads on the Rekenrek
2. The second child turns over a card and slides that number of beads on the Rekenrek
3. Find the difference between the two numbers.
4. Children record in their own way (or use the Rekenrek recording paper).

Appendix - page 5

Subtraction - One less, one more

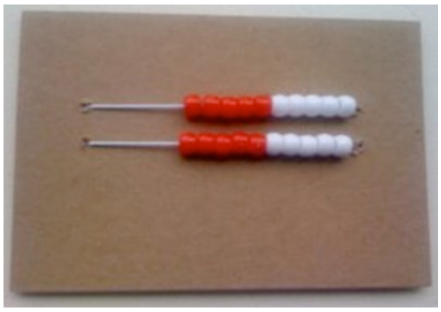
Using ‘tools to think with’ - Rekenrek

Subtraction

Take away Difference

The Rekenrek

Here are some rules about using a Rekenrek:



The beads are not counted, the children are encouraged to subitise (to know how many without counting).

The starting position should show all beads pushed to the far right.

Children enter the number by sliding beads to the left in a **one-push** motion.

Using one row on the Rekenrek - the other row is covered with paper or a cloth

One less, One more

The Rekenrek allows children to develop their conceptual understanding of addition and subtraction at the same time. They compose and decompose numbers, developing fluency with understanding.

Children will know how many without counting because during their experience of using the Rekenrek they will have become fluent in showing a given number in **one push.**

Develop this so that children can say how many beads there would be if there was one less. Ensure that you do not translate the meaning of less for the children by giving the instruction to “take away one”. Children need to learn and become fluent with the meaning of less.

Ask children to say what have they done in a sentence - “One less than seven is six”. This ensures that the children are not just responding to the word less but, that they are also building their mathematical vocabulary bank so that they use and apply this new concept through doing and saying.

Appendix - page 4

Subtraction - Difference

Using 'tools to think with' - Cuisenaire

Subtraction

Take away **Difference**

Cuisenaire

Children need to recognise all the rods by sight and by touch in order to be able to use them for developing their conceptual understanding.

Spin it, find it, spin it, find it, compare and find the difference…

(a game for players)

Resources: Cuisenaire rods, 0 - 10 or 5 - 15 spinner (appendix)

*Note: **No counting** - children must find the correct Cuisenaire rods, compare them and identify the difference without counting, by attending to the size of the space and identifying the rod that would fit in that space*

1. Spin the spinner and find the correct rod.
2. Spin the spinner again and find the correct rod.
3. Compare the two rods and say the difference without counting.
4. Say and write the number sentence using words.

There is a difference of 2 between 9 and 7

The difference between 9 and 7 is 2

5. Repeat at least 10 times

Describe what you see…

encourage children to describe what they see in other ways:

9 is 2 more than 7

7 is 2 less than 9

Negative Number Strategy

Subtraction

Take away **Difference**

Use nrich activity called ***Tug of War 5897*** to develop conceptual understanding of negative numbers

Eighty subtract thirty equals fifty

One subtract 7 equals negative 6

$$\begin{array}{rcc} & 8 & 1 \\ - & 3 & 7 \\ \hline \end{array}$$

50 - 6 = 44

$$\begin{array}{rcc} & 9 & 3 \\ - & 4 & 7 \\ \hline \end{array}$$

50 - 4 = 46

$$\begin{array}{rcc} & 7 & 2 \\ - & 3 & 4 \\ \hline \end{array}$$

40 - 2 = 38

$$\begin{array}{rcc} & 8 & 5 \\ - & 3 & 8 \\ \hline \end{array}$$

50 - 3 = 47

$$\begin{array}{rccc} & 1 & 3 & 3 \\ - & & 7 & 8 \\ \hline \end{array}$$

100 - 40 - 5 = 55

$$\begin{array}{rccc} & 3 & 5 & 1 \\ - & 2 & 2 & 8 \\ \hline \end{array}$$

100 + 30 - 7 = 123

$$\begin{array}{rccc} & 4 & 5 & 6 \\ - & 2 & 8 & 7 \\ \hline \end{array}$$

200 - 30 - 1 = 169

2 0 2 7

- 1 6 5 8

1000 - 600 - 30 - 1 = 369

Difference ITP

This ITP allows you to compare two rows of beads and to analyse the calculations they can represent.

It can be used to promote the language of addition and subtraction, particularly the interpretation of difference.

Select the number of yellow and white beads to view (the maximum is 30)
Click on the numbers to make the beads appear on screen.
You can move each row of beads up and down the screen at any point.

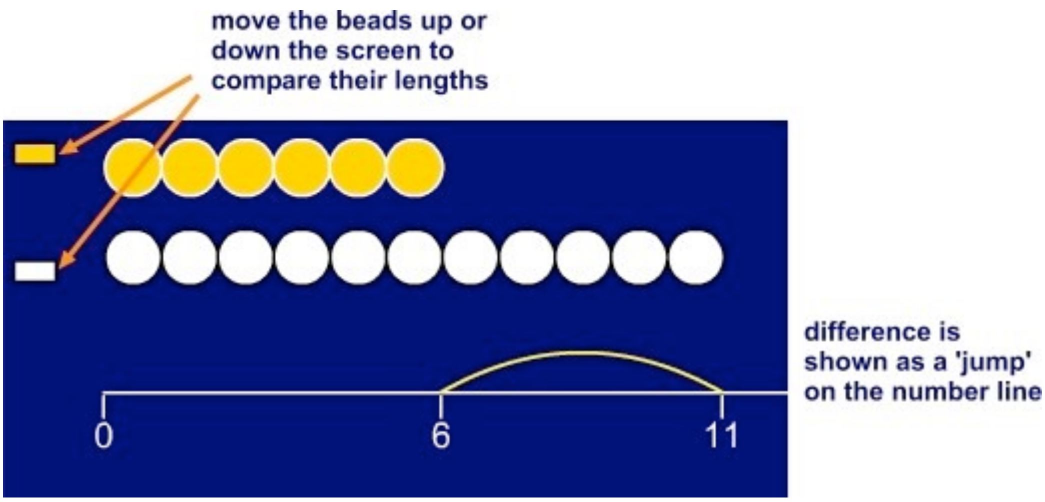
Click the play button to start each stage of the animation. The sequence is:

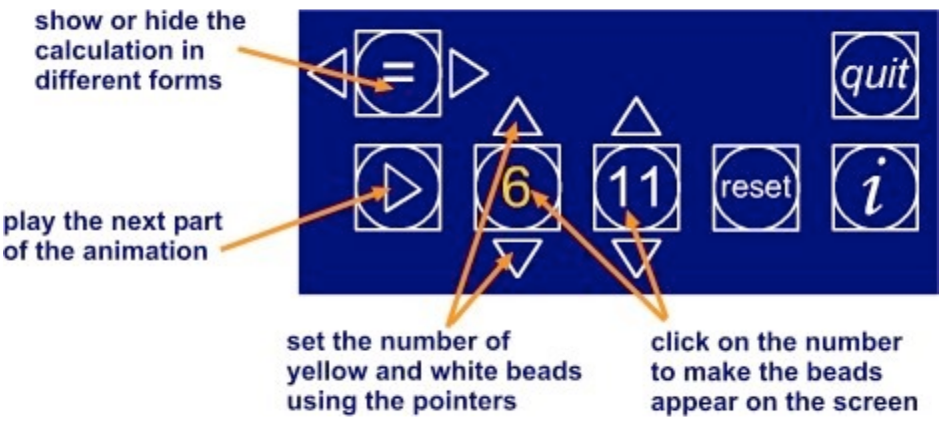
The top yellow bead line moves down until it is in line with the white bead line. The shorter line will be on top so that you can compare both bead lines.

A number line representing the yellow bead line appears.

A number line representing the white bead line appears.

The number lines merge and the difference is shown as a ‘jump’.





Making decisions about subtraction

Identify which of these calculation strategies is the most efficient and justify why.

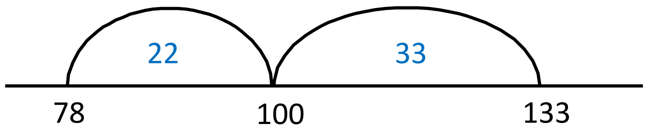
Cut into separate calculations, discuss and sort.

Subtraction

Take away **Difference**

Counting up on a number line

133 - 78



Decomposition

Crossing the tens and also the hundreds boundary

$$\begin{array}{rrrr} & & {}^{12} & {}^{1} \\ & \cancel{1} & \cancel{3} & 3 \\ - & & 7 & 8 \\ \hline & & 5 & 5 \\ \hline \end{array}$$

Negative number strategy

$$\begin{array}{rrrr} & 1 & 3 & 3 \\ - & & 7 & 8 \\ \hline \end{array}$$

$\underline{100 - 40 - 5} = 55$

The Same Difference

Round up the subtrahend

$$\begin{array}{rcr} 133 & - & 78 \\ +22 & & +22 \\ 155 & - & 100 \end{array}$$

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Making decisions about subtraction

Identify which of these calculation strategies is the most efficient and justify why.

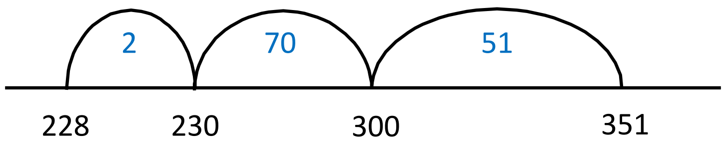
Cut into separate calculations, discuss and sort.

Subtraction

Take away   **Difference**

Counting up on a number line

351 - 228



Decomposition

Crossing the tens boundary but, **not** the hundreds boundary

$$\begin{array}{r} 3\ \overset{4}{\cancel{5}}\ {}^{1}1 \\ -\ 2\ \ 2\ \ 8 \\ \hline 1\ \ 2\ \ 3 \\ \hline \end{array}$$

Negative number strategy

$$\begin{array}{r} 3\ \ 5\ \ 1 \\ -\ 2\ \ 2\ \ 8 \\ \hline 100 + 30 - 7 \\ \hline \end{array} = 123$$

The Same Difference

Round up the subtrahend

$$351 - 228$$

$$+2 \qquad +2$$

$$353 - 230 = 123$$

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Making decisions about subtraction

Identify which of these calculation strategies is the most efficient and justify why.

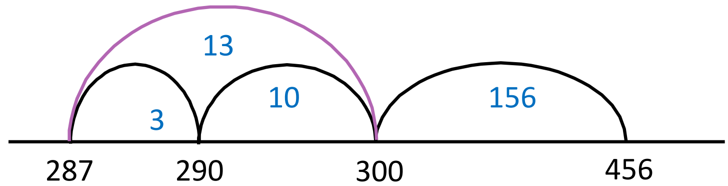
Cut into separate calculations, discuss and sort.

Subtraction

Take away **Difference**

Counting up on a number line

456 - 287



Decomposition

Crossing the tens boundary and also the hundreds boundary.

$$\begin{array}{r} {}^{14}\quad{}^{1} \\ \cancel{4}\;\cancel{5}\;6 \\ -\,2\;8\;7 \\ \hline 1\;6\;9 \\ \hline \end{array}$$

Negative number strategy

$$\begin{array}{r} 4\;\;5\;\;6 \\ -\,2\;\;8\;\;7 \\ \hline 200 - 30 - 1 \\ \hline \end{array} = 169$$

The Same Difference

Round up the subtrahend

$$\begin{array}{ccc} 456 & - & 287 \\ +13 & & +13 \\ 469 & - & 300 = 169 \end{array}$$

Making decisions about subtraction

Identify which of these calculation strategies is the most efficient and justify why.

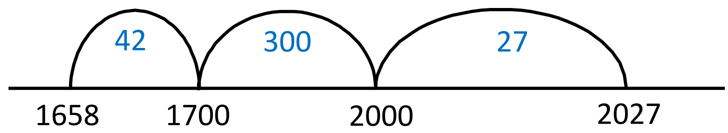
Cut into separate calculations, discuss and sort.

Subtraction

Take away ↔ **Difference**

2027 - 1658

Counting up on a number line



Decomposition

Crossing the tens boundary, the hundreds boundary, also the thousands boundary

1   1 9   1 1   1

$\cancel{2}$ $\cancel{0}$ $\cancel{2}$ 7

\- 1 6 5 8

3 6 9

Negative number model

2 0 2 7

\- 1 6 5 8

1000 - 600 - 30 - 1 = 369

The Same Difference

Round up the subtrahend

2027 - 1658

+42   +42

2069 - 1700

+300   +300

2369 - 2000 = 369

Subtraction

Which strategy and why?

Subtraction

Take away **Difference**

$$17 - 9 = 8$$

minuend - subtrahend = difference

10,000 - 10

249 - 99

5008 - 2994

136 - 84

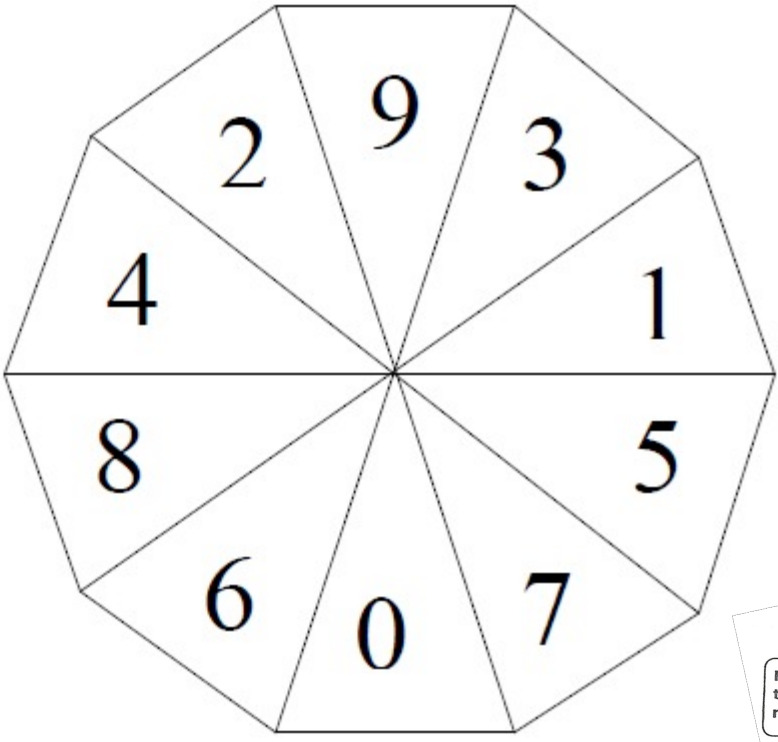
709 - 198

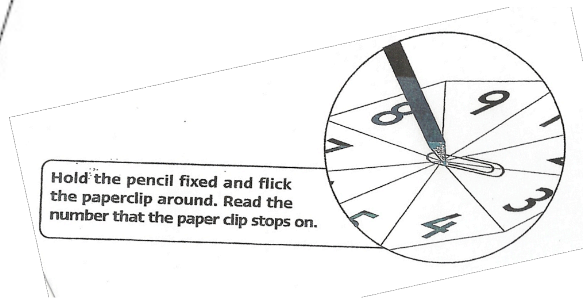
Resources

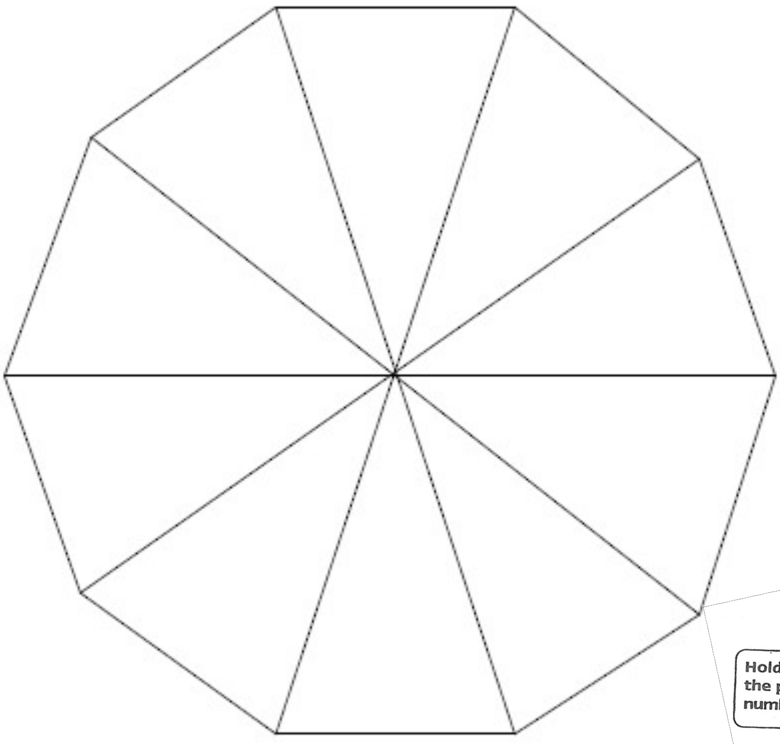
Rekenrek recording frame

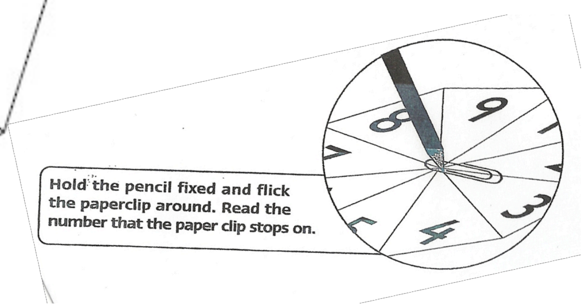
Spinners for games Blank

Spinners


9
3
1
5
7
0
6
8
4
2


Hold the pencil fixed and flick the paperclip around. Read the number that the paper clip stops on.




Hold the pencil fixed and flick the paperclip around. Read the number that the paper clip stops on.